# Hong Kong Housing Market Theory and Evidence

A Thesis
Submitted to the Division of Economics
of the Graduate School of
the Chinese University of Hong Kong
in Partial Fulfillment of the Requirements
for the Degree of
Master of Philosophy

Department of Economics

by

Wong Yim Chun

October 1998

# 摘要

本論文係由三個獨立部份組成，旨於探討房屋市場。

第一章《通貨膨脹、房屋價格及經濟增長》之目的是在理論層面上，闡釋房屋價格相對於整體經濟增長而言不斷上升之理由。在本章，作者建立了一個動態模型；同時，現金以預付現金 (cash-in-advance) 的方法被引入此模型中。以此模型為基礎，作者建立一個物價指數；在與廣泛地被利用的相對物價指數比較下，我們發現常用的相對物價指數存在著謬誤。

第二章為《香港的自然空置率》。本章旨在測定香港不同類別的房屋的自然空置率。利用 Gabriel and Nothaft (1988) 的方法，配上 1983 年至 1996 年香港的數據，我們確定了空置率與租金的上升速率有著反面的關係。同時，我們亦發現小型單位的自然空置率與大型單位自然空置率有顯著的差異。

在第三章《香港房屋市場的動態》中，作者計算了房屋市場的變數與主要經濟變數的相關系數，希望此舉能裨益於未來的房屋市場研究。

## Abstract

The three individual chapters that comprise this thesis are about property market. In the first chapter "Inflation, Housing Price, and Economic Growth", a dynamic model with money introduced by cash-in-advance methodology is built in order to give a theoretical explanation on why the housing price can keep rising relative to the economic growth of the whole economy. Based on the model, a price index is constructed, we find that the commonly used relative price index is subject to fallacy.

In the second chapter, natural vacancy rates of different types of houses in Hong Kong are determined. Based on the methodology utilized in Gabriel and Nothaft (1988), using Hong Kong data from 1983 to 1996, we confirm the view that there is an inverse relation between vacancy rate and growth of rent. Further, we find that natural vacancy rates of small flats and large flats differ significantly.

Correlations between property market variables and main economic variables are computed in the third chapter. We hope that this can benefit research on property market.

## Acknowledgments

Many thanks to my thesis committee, Professor Charles Leung Ka Yui, Professor Chou Win Lin, and Professor Sunny Kwong Kai Sun. Apart from ordinary guidance provided, Professor Leung taught me how to survive under the extreme situation which will be invaluable in my future career; Professor Chou was really helpful, she gave useful suggestion on the content and the presentation of this thesis; Professor Kwong raised interesting questions during my preliminary presentation, it made me rethink the content of the work. Of course, I remain solely responsible for any errors.

Special thanks to Mr. Lee Chi Kau who gave me technical support. I am grateful to the helpful discussion with Miss Sze Kin Wan and Mr. Li Yan.

Last but not least, I would like to thank all my classmates in the Department of Economics for they brought me joy and happiness during the two years' M.Phil study.

# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF CHARTS

**Chapter 4:**

# LIST OF APPENDICES

# Chapter 1

# Introduction

'Clothing, eating, housing and travelling' are traditionally regarded as basic elements of living. For Hong Kong people, the one which is the most difficult to be satisfied may be the third one: housing. It is well known that Hong Kong is in short of land. Although there is about 1095km$^2$ of land in Hong Kong (of which 80km$^2$ is from Hong Kong Island; 47km$^2$ from Kowloon; 794km$^2$ from New Territories; and 174 km$^2$ from the outlying islands), it is fragmented into more than 200 islands. With her 6.3 millions residence, Hong Kong becomes a very dense place to live in. Consequentially, housing is always a problem of the Hongkongers.[1]

House possesses a dual characteristic: it can be regarded either as a consumption good or an investment good, or both. Everyone living in a house is consuming the services derived from the house: a place for sheltering, gathering, and rest. The consumption nature of house is obvious. Being an investment good, the performance of house is very remarkable: housing price index of Hong Kong from 1984 to 1994 has an average growth rate of 21.15 percent; in other word, the real housing price of Hong Kong has risen nearly seven times. While, of the same period, the consumer price index A (CPIA) has an average growth of 7.61 percent; the growth rate of housing price is substantially higher than that of the general price index. On top of that, the

[1]Data obtain from *Hong Kong 1997*.

rental index during the same period rase from 54 to 170; which means an average growth of 12.15 percent.[2] If a person who buys a flat for rent, he/she does not only enjoy the increase in value of the asset but also receive a rental income which grows at a faster pace than the general price. The rate of return by investing in the housing market is really attractive.

The dual characteristic of house keeps the housing market in a dynamic state and makes it interesting to investigate.

Consisting of three independent chapters, this thesis is about the housing market of Hong Kong. A dynamic model with cash-in-advance constraint is constructed in chapter one. The objective of the model is to explain the commonly seen phenomenon: the ever-increasing housing price. A price index is also constructed in order to contrast with the commonly used relative price index. In chapter two, a pooled cross-section time-series econometric model is built in order to determine the natural vacancy rates of different types of houses in Hong Kong. Dynamics of Hong Kong housing market will be presented in chapter three.

[2] Figures here are computed from data obtained in *Hong Kong Monthly Digest of Statistics*.

# Chapter 2

# Inflation, Housing Price, and Economic Growth

## 2.1 Introduction

As mentioned in the introduction, the housing price index of Hong Kong has an average growth rate of 21.15 percent from 1984 to 1994; while, during the same period, the rental index has an average growth rate of 12.15 percent. Comparing with the nominal GDP growth (14.70 percent) and growth of CPIA (7.61 percent) of the same period, the growth of housing price is really shocking.[1] However, what we learn from macroeconomics class suggests that, at the steady state, all real variables should grow in the same pace; why do the housing prices in Hong Kong grow faster? Actually, we can find that citizens in major cities all over the world are suffering from high-rising housing prices[2]. It is hard to believe that the ever-increasing housing price is a particular, temporary phenomenon.

A dynamic model is set up in this chapter. The goal of the model is to explain why the housing price can keep growing, relative to the whole economy, even at the

[1]Figures here are computed from data obtained in the *Hong Kong Monthly Digest of Statistics*.
[2]Leung (1998) has done an extensive research on this.

steady state. Money is introduced in the model through cash-in-advance constraint in order to examine whether money matters in such a economy. As housing price is the main concern of this chapter, two price indices, based on the settings of the model, are presented for comparison.

This chapter proceeds as follows: section 2 presents the basic model, and a description of the operating environment of the economy. Results of different scenarios of the model due to variations of the cash-in-advance constraints are derived in section 3. The comparison of the price indices is made in section 4. A short summary concludes this chapter.

## 2.2 Basic Model

Consider an economy in which the individual is maximizing his/her expected lifetime utility. What matter in the individual's utility function are the level of consumption and the stock of housing the individual possesses, as given by:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t, h_t\right)\right], 0<\beta<1, \tag{2.1}$$

The utility function adopted here is a special case that considered in Greenwood and Hercowitz (1991):

$$U(c_t, h_t) = \ln c_t + \omega \ln h_t,$$

where $c_t$ is the non-durable consumption, $h_t$ is the stock of housing and $\omega > 0$ is a preference parameter. The utility function satisfies the balanced growth condition shown in King, Plosser and Rebelo (1988).

At the beginning of each period $t$, the individual has $h_t$ units of houses, $k_t$ units of capital pre-installed in the non-durable producing sector, $k_t^h$ units of capital pre-installed in the house producing sector, $l_t$ units of land, $m_{t-1}$ units of cash, and $\tau_t$ units of cash transferred from the government. The dynamic programming problem for a representative individual is:

$$\begin{aligned} V(h_t, h_t^m, k_t, k_t^h, l_t, m_t) &= \max_{c_t, k_{t+1}, k_{t+1}^h, m_{t+1}, l_{t+1}, h_{t+1}, h_{t+1}^m} \ln c_t + \omega \ln h_t \\ &\quad +\beta V(h_{t+1}, h_{t+1}^m, k_{t+1}, k_{t+1}^h, l_{t+1}, m_{t+1}). \end{aligned} \tag{2.2}$$

An agent is facing three constraints. First, it is the budget constraint:

$$c_t + k_{t+1} + k_{t+1}^h + p_t^h h_t^m + p_t^l (l_{t+1} - l_t) + \frac{m_t^d}{P_t} \leq Ak_t + \frac{m_{t-1} + \tau_t}{P_t}. \tag{2.3}$$

The expression (2.3) implies that the individual allocates the total output $Ak_t$ and the money he/she holds $\frac{m_{t-1} + \tau_t}{P_t}$ for consumption $c_t$, investment in both goods producing sector and construction sector respectively $k_{t+1}$ and $k_{t+1}^h$, the extra land $(l_{t+1} - l_t)$ at the unit price $p_t^l$, new houses $h_t^m$ at the unit price $p_t^h$ from the market, and the money balances that the individual wants to carry over to the next period $m_t^d$ in order to finance his/her purchases at time $t + 1$. $P_t$ here means the relative price of money to the consumption/investment good.

The second one is the cash-in-advance constraint. Imagine an economy with a large number of individuals, each of them produces $y$ units of consumption goods but he/she cannot consume what he /she produces; rather, the individual has to buy consumption goods from the others with cash and sell what he/she produced to the others to get the money. The individual possesses some money at the beginning of each period. What he/she has to do in each period is (1) to use all or part of the money that he/she initially possesses to buy consumption goods and (2) to produce consumption goods and sell them to the others. The next period's initial money balances will be the sum of the unspent money, the recipients from the sale of goods, and the transfer from the government. The story goes on and on in this manner.

As the form of the cash-in-advance constraint depends on the underlined assumptions of different cases, we will leave the discussion of this constraint in the corresponding subsections.

The last constraint is the 'flow of house',

$$h_{t+1} \leq h_t + h_t^m + (k_t^h)^\alpha (l_t)^{1-\alpha}. \tag{2.4}$$

Equation (2.4) tells us that the next period's stock of housing is determined by three elements: the existing housing stock $h_t$, the amount of house bought from the market $h_t^m$, and those constructed by the individual himself/herself $(k_t^h)^\alpha (l_t)^{1-\alpha}$. For simplicity, the depreciation rates of houses and land are assumed to be zero; and that of the capital engaged in the construction sector is 100%. Notice that, contrary to the goods producing sector, the construction sector takes time to produce. That is, this period's production of house contribute to next period's stock.

Money is introduced in the framework by cash-in-advance methodology. Five different scenarios are examined, namely, (1) all transactions are subject to cash-in-advance constraint, (2) only consumption is subject to cash-in-advance constraint, (3) both consumption and capital are subject to cash-in-advance constraint, (4) consumption, capital, and capital for house building are subject to cash-in-advance constraint, and finally (5) consumption, capital, capital for house building, and purchase for house are subject to cash-in-advance constraint. Case (1) and case (2) will be presented in the following while the rest of the cases will be discussed in the appendices.

## 2.3 Results

### 2.3.1 Case 1: All transactions are subject to cash-in-advance constraint

In the present case, the individual is maximizing the dynamic programming problem (2.2),

$$\begin{aligned} V(h_t, h_t^m, k_t, k_t^h, l_t, m_t) &= \max_{c_t, k_{t+1}, k_{t+1}^h, m_{t+1}, l_{t+1}, h_{t+1}, h_{t+1}^m} \ln c_t + \omega \ln h_t \\ &\quad + \beta V(h_{t+1}, h_{t+1}^m, k_{t+1}, k_{t+1}^h, l_{t+1}, m_{t+1}) \end{aligned}$$

s.t.

$$c_t + k_{t+1} + k^h_{t+1} + p^h_t h^m_t + p^l_t(l_{t+1} - l_t) + \frac{m^d_t}{P_t} \leq Ak_t + \frac{m_{t-1} + \tau_t}{P_t},$$

$$c_t + k_{t+1} + k^h_{t+1} + p^h_t h^m_t + p^l_t(l_{t+1} - l_t) \leq \frac{m_{t-1} + \tau_t}{P_t}, \tag{2.5}$$

and

$$h_{t+1} \leq h_t + h^m_t + (k^h_t)^\alpha (l_t)^{1-\alpha}.$$

The cash-in-advance constraint (2.5) here means that no matter what the individual is going to purchase, cash is needed in advance. The total amount of cash that the individual has at the period $t$ is that he/she carried over from the last period $t-1$ and that transferred by the government at the beginning of period $t$; therefore, the value of $\frac{m_{t-1} + \tau_t}{P_t}$ must be greater than or equal to the value of total purchase at period $t$.

At the equilibrium, left hand side and right hand side of the cash-in-advance constraint will be equal, i.e. money carried over from the previous period will be completely used up in this period; budget constraint (2.3) and 'flow of house' constraint (2.4) will hold in equality as well. Therefore, (2.3), (2.5) and (2.4) become:

$$\frac{m^d_t}{P_t} = Ak_t, \tag{2.6}$$

$$c_t + k_{t+1} + k^h_{t+1} + p^h_t h^m_t + p^l_t(l_{t+1} - l_t) = \frac{m_t}{P_t} \tag{2.7}$$

and

$$h_{t+1} = h_t + h^m_t + (k^h_t)^\alpha (l_t)^{1-\alpha}. \tag{2.8}$$

With $\lambda_{1,t}, \lambda_{2,t}$ and $\lambda_{3,t}$ be the multipliers of the constraints (2.6), (2.7) and (2.8), respectively. We can easily obtain the first order conditions:

$$\frac{1}{c_t} = \lambda_{2,t}, \tag{2.9}$$

$$\lambda_{3,t} = \beta(\frac{\omega}{h_{t+1}} + \lambda_{3,t+1}), \tag{2.10}$$

$$\lambda_{2,t}p_t^h = \lambda_{3,t}, \tag{2.11}$$

$$\lambda_{2,t}p_t^l = \beta[\lambda_{2,t+1}p_{t+1}^l + \lambda_{3,t+1}(1-\alpha)(k_{t+1}^h)^\alpha(l_{t+1})^{-\alpha}], \tag{2.12}$$

$$\lambda_{2,t} = \alpha\beta\lambda_{3,t+1}(k_{t+1}^h)^{\alpha-1}(l_{t+1})^{1-\alpha}, \tag{2.13}$$

$$\lambda_{2,t} = A\beta\lambda_{1,t+1}, \tag{2.14}$$

and

$$\frac{\lambda_{1,t}}{P_t} = \beta\frac{\lambda_{2,t+1}}{P_{t+1}}. \tag{2.15}$$

Now, we turn to solve the whole system; that is, (1) to find all the initial values of the endogenous variables in terms of the exogenous variables and constants and (2) to find the growth factors of all variables, and (3) to deduce the initial conditions such that the economy will finally on the balance growth path. Notice that, at the equilibrium,

$$l_t - l_{t-1} = h_t^m = 0.$$

This is because when everyone wants to increase his/her land possession and buy houses in the market, no one can buy anything ultimately. $l_t$ is normalize to 1 for simplicity.

By solving (2.10) iteratively, we have:

$$\lambda_{3,t} = \frac{\beta\omega}{h_t(g_h - \beta)} \tag{2.16}$$

If we define the growth factor of variable $i$ as

$$g_i \equiv \frac{i_{t+1}}{i_t}, \tag{2.17}$$

and the growth of money supply as

$$\mu \equiv \frac{m_{t+1}}{m_t} \tag{2.18}$$

we will have the following lemma:

**Lemma 1** *If the growth factor and the growth of money supply are defined as (2.17) and (2.18), respectively, combining (2.16) with the FOCs, then:*

1. *The growth of consumption, the growth of capital in construction sector, the growth of capital in producing sector, and the growth of price of land will all be equal:*

$$g \equiv g_c = g_{k^h} = g_k = g_{p^l}.$$

*And we call $g$ the common growth rate.*

2. *The growth rate of houses equals to the common growth rate to the power $\alpha$ :*

$$g_h = g^{\alpha}$$

3. *The growth rate of the housing price equals to the common growth rate to the power $1-\alpha$:*

$$g_{p_h} = g^{1-\alpha}$$

4. *The growth rate of the relative price of money to the consumption good is:*

$$g_p = A\beta^2 \cdot \frac{1}{g^2}.$$

The proof of the lemma is in the appendices.

With the lemma above, we have the following proposition:

**Proposition 2** *The common growth equals to:*

$$g = \frac{A\beta^2}{\mu}.$$

The proof of the proposition is in the appendices.

The initial values of the endogenous variables, $c_t, p_t^l, p_t^h$ and $P_0$ are :

$$c_0 = \frac{A}{\alpha\omega\mu}(\frac{g^{\alpha}-\beta}{g^{\alpha}-1})k_0^h,$$

$$p_0^l = \left[\frac{A\beta^2(1-\alpha)}{\alpha\mu(1-\beta)}\right] k_0^h,$$

$$p_0^h = \frac{A\beta}{\alpha\mu}(k_0^h)^{1-\alpha},$$

and

$$P_0 = \frac{m_0}{Ak_0}.$$

**Lemma 3** *The initial conditions needed for the economy to reach the balanced growth path are:*

$$\xi_1 k_0 = k_0^h$$

*where* $\xi_1 = (A-g)/\left[\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)} + g\right]$ *and*

$$h_0 = \frac{(k_0^h)^\alpha}{g_h - 1}.$$

Note that the initial condition $h_0 = \frac{(k_0^h)^\alpha}{g_h - 1}$ is deduced from the 'flow of house' constraint (2.8) and this constraint will not be altered. Furthermore, from (2.8), we can immediately obtain

$$g_h = (g_{k^h})^\alpha$$

Therefore, this initial condition and the relationship between $g_h$ and $g_{k^h}$ are valid for all the cases that we are going to discuss. We will not explicitly state in the following content.

### 2.3.2 Case 2: only consumption is subject to cash-in-advance constraint

We are now going to examine the other extreme: only consumption is subjected to cash-in-advance constraint. The budget constraint and the constraint of 'flow of house' are exactly the same as those in the previous case. Since only consumption is subject to cash-in-advance constraint, money is used only if the individual wants to consume non-durable goods. The money carried over from the previous period plus the government transfer should be greater than or equal to the market value of the non-durable goods that the individual desires. Therefore, the cash-in-advance constraint for this problem is:

$$c_t \le \frac{m_{t-1} + \tau_t}{P_t}. \tag{2.19}$$

Similar to the first case, at steady state, all the constraints will hold in equality. Therefore, the constraints becomes:

$$k_{t+1} + k^h_{t+1} + p^h_t h^m_t + p^l_t(l_{t+1} - l_t) + \frac{m^d_t}{P_t} = Ak_t, \tag{2.20}$$

$$c_t = \frac{m_t}{P_t}, \tag{2.21}$$

and

$$h_{t+1} = h_t + h^m_t + (k^h_t)^\alpha (l_t)^{1-\alpha}.$$

The first order conditions are not difficult to derive:

$$\frac{1}{c_t} = \lambda_{2,t}, \tag{2.22}$$

$$\lambda_{3,t} = \beta[\frac{\omega}{h_{t+1}} + \lambda 47_{3,t+1}], \tag{2.23}$$

$$\lambda_{1,t} p^h_t = \lambda_{3,t}, \tag{2.24}$$

$$\lambda_{1,t} p^l_t = \beta[\lambda_{1,t+1} p^l_{t+1} + \lambda_{3,t+1}(1-\alpha)(k^h_{t+1})^\alpha (l_{t+1})^{-\alpha}], \tag{2.25}$$

$$\lambda_{1,t} = \alpha\beta\lambda_{3,t+1}(k^h_{t+1})^{\alpha-1}(l_{t+1})^{1-\alpha}, \tag{2.26}$$

$$\lambda_{1,t} = A\beta\lambda_{1,t+1}, \tag{2.27}$$

and

$$\frac{\lambda_{1,t}}{P_t} = \beta\frac{\lambda_{2,t+1}}{P_{t+1}}. \tag{2.28}$$

As (2.10) is the same as (2.23), this means the expression of $\lambda_{3,t}$ is the same as case 1:

$$\lambda_{3,t} = \frac{\beta\omega}{h_t(g_h - \beta)}.$$

**Lemma 4** *With $\lambda_{3,t}$ and the FOCs we have:*

1. *The growth of consumption, the growth of capital in producing sector, the growth of land price, and the growth of capital in construction sector are all equalized:*

$$g \equiv g_c = g_k = g_{p^l} = g_{k^h}$$

   *as a rule, $g$ is called the common growth rate.*

2. *The growth of housing price is equal to the common growth to the power $1-\alpha$,*

$$g_{p^h} = g^{1-\alpha}.$$

3. *The growth rate of the relative price of money to the consumption/investment good is:*

$$g_p = \frac{\mu}{g}.$$

With the above lemma. we have the following proposition:

**Proposition 5** *The common growth rate of this case is*

$$g = A\beta.$$

The proof of the above lemma and proposition is in the appendices.

With $l_t$ normalized to 1, the initial values of the endogenous variables are:

$$c_0 = \frac{A}{\alpha\omega\mu}(\frac{g^\alpha - \beta}{g^\alpha - 1})k_0^h,$$

$$p_0^l = \frac{A\beta}{\alpha}(\frac{1-\alpha}{1-\beta})k_0^h,$$

$$p_0^h = \frac{A}{\alpha}(k_0^h)^{1-\alpha}$$

and

$$P_0 = \left[\frac{\alpha\omega\mu(g^\alpha - 1)}{A(g^\alpha - \beta)}\right]\frac{m_0}{k_0^h}.$$

**Lemma 6** *The initial condition needed for the economy to reach the balanced growth path is*

$$\xi_2 k_0 = k_0^h$$

*where* $\xi_2 = (A-g)/\left[\dfrac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)} + g\right]$.

Results in all cases are summarized in table 2.1a and 2.1b for comparison.

*(Table 2.1a and 2.1b are about here)*

From the results obtained above, it is noticeable that the common growth rate of the real variables, $g$ is equal to $\dfrac{A\beta^2}{\mu}$ in case one (actually, the common growth rate of all the cases except the second case, namely, only consumption is subject to cash-in-advance constraint, is equal to $\dfrac{A\beta^2}{\mu}$. Please refer to table 2.1a) where $\mu$ is the growth factor of money supply; in order to keep the economy growing, i.e., $g > 1$, the increase in money supply cannot be 'too fast' or the technology of the economy cannot be 'too slow'. We may assume that

$$\frac{A\beta^2}{\mu} > 1; \tag{2.29}$$

while the common growth factor of the case two is $A\beta$. As money growth rate, $\mu$, is usually greater than 1 while the discount factor, $\beta$, is normally less than 1 in reality. Therefore we have

$$A\beta > \frac{A\beta^2}{\mu} > 1.$$

With the assumption (2.29), several results can be established:

**Proposition 7** *The housing stock grows slower than the other real variables.*

If we define $\gamma_h$ be the net growth rate of housing stock (the net growth rate of the other variables are defined analogously), $\gamma_h = g_h - 1$ and $\gamma = g - 1$, we have

$$\gamma_h \cong \alpha\gamma;$$

the net growth rate of housing stock is equal to the product of the share of capital in the construction industry, $\alpha\,(< 1)$ and the common net growth rate of the aggregate variables. This is not difficult to explain. As land is an essential input of the construction industry and the amount of land is fixed, the houses, as a result, cannot grow as fast as the rest of the economy. This phenomenon is reflected in the price of the houses—as presented in the next proposition.

**Proposition 8** *The relative price of housing is increasing over time, i.e.* $g_{p^h} > 1$.

As $g_{p^h} = g^{1-\alpha}$, similar to the previous proposition, we have

$$\gamma_{p^h} \cong (1-\alpha)\gamma;$$

the net growth of the housing price is the share of land engaged in the construction industry times the common net growth rate of the aggregate variables.

With this proposition, the relative price of houses is certainly growing slower than the economy.

**Proposition 9** *The relative price of land is increasing over time, i.e.* $g_{p^l} > 1$.

In fact, the net growth of the relative price of land is equal to the net growth of the economy: $\gamma_{p^l} = \gamma$.

**Corollary 10** *The ratio of the expenditure between non-durable consumption and housing acquisition* $\varpi_t \equiv \dfrac{c_t}{h_t \cdot p_t^h}$ *is constant over time.*

**Proof.** To show $\varpi_t$ is constant over time, it suffices to show that $\varpi_t = \varpi_{t+1}$ for all time $t$. Note that $\dfrac{\varpi_{t+1}}{\varpi_t} = \left(\dfrac{c_{t+1}}{h_{t+1} \cdot p_{t+1}^h}\right) / \left(\dfrac{c_t}{h_t \cdot p_t^h}\right) = \dfrac{g}{g_h \cdot g_{p^h}}$; by the results stated above, it is obvious that $\dfrac{\varpi_{t+1}}{\varpi_t}$ is equal to unity. ■

Note that the growth factor in case two is $A\beta$ while that of case one is $\dfrac{A\beta^2}{\mu}$. This means that when only consumption in an economy is subject to cash-in-advance constraint, the economy will grow at a faster pace than the one with more restrictive constraints. In other words, a more efficient financial market will be advantageous to an economy and this is paralleled to the conventional wisdom. With a smaller growth factor, it is clear that both growth of housing stock and growth of relative price of houses will be lower.

## 2.4 Price Index

In this section, we will discuss effect of the increasing housing price towards the price index. Conventionally, the price index, at time $t$, is formulated as the summation of the prices of goods weighted by their corresponding share of expenditure at time $t$,

$$\mathbb{P}_t = \sum_i (\text{share of expenditure})_{i,t}(\text{price})_{i,t}. \tag{2.30}$$

In the present model the share of expenditure, along the balanced growth path, will not vary. The change in price index is solely due to the variation of price.

In this economy, there are only two kinds of goods: the consumption/investment goods and houses. Production during year $t$ is $c_t, k_{t+1}$ and $k_{t+1}^h$ plus the houses newly produced $(k_t^h)^\alpha(l_t)^{1-\alpha}$.

The GDP of year $t$ in terms of consumption/investment goods, $y_t$, is

$$y_t = (c_t + k_{t+1} + k_{t+1}^h) + p_t^h h_t(g_h - 1),$$

with the price of consumption/investment goods normalized to unity (numeraire) and the relative price of newly produced houses be $p_t^h$. By (2.8), it is easy to show that $(k_t^h)^\alpha(l_t)^{1-\alpha} = h_t(g_h - 1)$. Therefore, the total value of the new houses, in terms of the consumption/investment goods, is $p_t^h h_t(g_h - 1)$. The share of consumption/investment and newly built houses are

$$\frac{(c_t + k_{t+1} + k_{t+1}^h)}{y_t} \text{ and } \frac{p_t^h h_t(g_h - 1)}{y_t},$$

respectively.

The nominal price of consumption/investment goods is $P_t$ while newly built houses is $P_t \cdot p_t^h$. By (2.30), the price index at period $t$ is

$$\mathbb{P}_t = \left[\frac{(c_t + k_{t+1} + k_{t+1}^h)}{y_t}\right](P_t) + \left[\frac{p_t^h h_t(g_h - 1)}{y_t}\right](P_t \cdot p_t^h).$$

By (2.3), at equilibrium, we have

$$y_t = Ak_t + p_t^h h_t(g_h - 1); \tag{2.31}$$

thus,

$$\mathbb{P}_t = P_t\left[(1 - s_h) + s_h p_t^h\right]$$

where $1 > s_h > 0$ is the value of newly produced houses as the share of the total GDP at period $t$,

$$s_h = \frac{p_t^h h_t(g_h - 1)}{y_t}.$$

Obviously, $s_h$ is potentially time-varying. However, from the results in the previous section, we have $g_h = g^\alpha$, $g_{p^h} = g^{1-\alpha}$, and $g_k = g$. Therefore, we can deduce that both $p_t^h h_t$ and $k_t$ grow at the same speed $g$. By (2.31), GDP at period $t$, $y_t$, is a linear combination of $k_t$ and $p_t^h h_t$ without a constant term; so, $y_t$ grows as the same rate of $k_t$ and therefore $p_t^h h_t$. As a result, $s_h$ is a constant along the balanced growth path.[3]

Since both $p_t^h$ and $P_t$ is increasing over time, that is $g_{p^h}, g_P > 1$,[4] we assume that

$$p_t^h,\ P_t > 1 \Rightarrow \mathbb{P}_t > 1. \tag{2.32}$$

Let $\pi_t$ be the net growth rate of the price index $\mathbb{P}_t$,

$$\pi_t = \frac{\mathbb{P}_{t+1}}{\mathbb{P}_t} - 1, \tag{2.33}$$

we will have the following proposition.

[3] As $g_h = g^\alpha$, $g_{p^h} = g^{1-\alpha}$, and $g_k = g$ are the common results of all the scenarios that we have examined in this paper, the conclusion that $s_h$ is constant holds in all cases.

[4] For $g_P > 1$, we have to assume that $\mu > g$. In other words, for case 2, we assume $\mu > A\beta$; for the other cases, we assume $\mu > \beta\sqrt{A}$.

**Proposition 11** *If the inflation rate $\pi_t$ is defined as (2.33) and the assumption (2.32) holds, we have*

1. *The inflation rate will be lower than the net growth rate of nominal housing price, that is,*
$$\pi_t < g_p g_{p^h} - 1.$$
2. *When the relative price of house increases, the inflation will increase as well,*
$$\frac{\partial \pi_t}{\partial p_t^h} > 0.$$
3. *Therefore, asymptotically, the inflation rate will be equal to net growth rate of the nominal housing price,*
$$\pi_t \rightarrow (g_p g_{p^h} - 1),\ as\, p_t^h \rightarrow \infty.$$

The proof of the above proposition is in the appendices.

The rationale of the proposition is straightforward. There are only two goods in the economy: the investment/consumption good with price $P_t$ and house with price $P_t p_t^h$. Since the price index, $\mathbb{P}_t$, is the weighted sum of $P_t$ and $P_t p_t^h$, the net growth rate of $\mathbb{P}_t$, that is $\pi_t$, should lie between the net growth rate of $P_t$ and $P_t p_t^h$. As the price of house tends to infinity, the growth rate of housing price will dominate the growth rate of the price index.

Now we are going to compare the general price index with the commonly used 'relative price index' in nominal terms:

$$p_{h,t}^n \equiv \frac{P_t p_t^h}{\mathbb{P}_t} = \frac{p_t^h}{(1 - s_h) + s_h p_t^h} \tag{2.34}$$

we have the following proposition:

**Proposition 12** *If the relative price index is defined as (2.34) and (2.32) holds, then*

1. *$p^n_{h.t}$ underestimates the real housing price, $p^h_t$,*

$$p^n_{h,t} < p^h_t$$

2. *As the housing price increases over time, the relative price will increase as well,*

$$\frac{\partial p^n_{h,t}}{\partial p^h_t} > 0.$$

3. *Asymptotically, the nominal price index will tend to the inverse of the share of expenditure in housing stock,*

$$as\, p^h_t \to \infty,\ then\, p^n_{h,t} \to \frac{1}{s_h}.$$

4. *The net growth rate of the relative price index is lower than the true growth rate of the housing price,*

$$\pi^n_{h,t} \equiv \frac{p^n_{h,t+1}}{p^n_{h,t}} - 1 < (g_{p^h} - 1).$$

5. *As the real housing price increase, the net growth rate of the relative price will decrease,*

$$\frac{\partial \pi^n_{h,t}}{\partial p^h_t} < 0$$

6. *Asymptotically, the growth rate of the relative price index will converge to zero,*

$$as\, p^h_t \to \infty,\ then\, \pi^n_{h,t} \to 0.$$

The result is interesting. As the real price of housing stock grows over time, it will dominate the 'real' general price index, $(1 - s_h) + s_h p^h_t$. Therefore, the relative price index becomes the ratio of the real price of house and the value of expenditure on housing stock. The relative price will converge to a constant $\frac{1}{s_h}$ and the net growth rate of the relative price index will converge to zero. Needless to say, the relative price index is very misleading and should be reconsidered while using.

## 2.5 Conclusion

In the first chapter of this thesis, a dynamic model with cash-in-advance constraint is constructed. The finding is not surprising: as quantity of land is limited, the price of house can grow faster than the whole economy. One may argue that the stock of land can be increased by reclamation. However, in the first place, this is not practical for non-coastal cities; secondly, even for a coastal city like Hong Kong, land increased through reclamation since 1851 is only $60\text{km}^2$.[5] Relative to her population growth, the increase in land by reclamation does not help much in easing the housing problem.

If the limitation of land is the reason for the high-rising of housing price, the development of rural area seems to be a method to solve the problem. As lots of the areas of New Territories and Lantau are still unused, development of these areas is necessary to solve the housing problem.

Based on the dynamic model above, we presented two price indices. One is constructed on the ground of the definition of price index; the other is the commonly adopted one. We find that, as time tends to infinity, the latter one does not make any sense at all. Therefore, we have to place extra attention when using the price index.

[5]Data from *Hong Kong 1997*.

**Table 2.1**: Growth Rates of Real Variables in Different Cases

| | case 1 | case 2 | case 3 | case 4 | case 5 |
|---|---|---|---|---|---|
| $g_c$ | $\frac{A\beta^2}{\mu}$ | $A\beta$ | $\frac{A\beta^2}{\mu}$ | $\frac{A\beta^2}{\mu}$ | $\frac{A\beta^2}{\mu}$ |
| $g_k$ | $\frac{A\beta^2}{\mu}$ | $A\beta$ | $\frac{A\beta^2}{\mu}$ | $\frac{A\beta^2}{\mu}$ | $\frac{A\beta^2}{\mu}$ |
| $g_{k^h}$ | $\frac{A\beta^2}{\mu}$ | $A\beta$ | $\frac{A\beta^2}{\mu}$ | $\frac{A\beta^2}{\mu}$ | $\frac{A\beta^2}{\mu}$ |
| $g_{p^l}$ | $\frac{A\beta^2}{\mu}$ | $A\beta$ | $\frac{A\beta^2}{\mu}$ | $\frac{A\beta^2}{\mu}$ | $\frac{A\beta^2}{\mu}$ |
| $g_{p^h}$ | $\left(\frac{A\beta^2}{\mu}\right)^{1-\alpha}$ | $(A\beta)^{1-\alpha}$ | $\left(\frac{A\beta^2}{\mu}\right)^{1-\alpha}$ | $\left(\frac{A\beta^2}{\mu}\right)^{1-\alpha}$ | $\left(\frac{A\beta^2}{\mu}\right)^{1-\alpha}$ |
| $g_P$ | $\frac{\mu^2}{A\beta^2}$ | $\frac{\mu}{A\beta}$ | $\frac{\mu^2}{A\beta^2}$ | $\frac{\mu^2}{A\beta^2}$ | $\frac{\mu^2}{A\beta^2}$ |
| $g_h$ | $\left(\frac{A\beta^2}{\mu}\right)^{\alpha}$ | $(A\beta)^{\alpha}$ | $\left(\frac{A\beta^2}{\mu}\right)^{\alpha}$ | $\left(\frac{A\beta^2}{\mu}\right)^{\alpha}$ | $\left(\frac{A\beta^2}{\mu}\right)^{\alpha}$ |

**Table 2.2**: Initial Values of the Endogenous Variables in Different Cases:

| | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|---|---|---|---|---|---|
| $c_0$ | $\frac{A}{\alpha\omega\mu}(\frac{g^\alpha-\beta}{g^\alpha-1})k_0^h$ | $\frac{A}{\alpha\omega\mu}(\frac{g^\alpha-\beta}{g^\alpha-1})k_0^h$ | $\frac{A\beta}{\alpha\omega\mu^2}(\frac{g^\alpha-\beta}{g^\alpha-1})k_0^h$ | $\frac{A}{\alpha\omega\mu}(\frac{g^\alpha-\beta}{g^\alpha-1})k_0^h$ | $\frac{A}{\alpha\omega\mu}(\frac{g^\alpha-\beta}{g^\alpha-1})k_0^h$ |
| $P_0$ | $\frac{m_0}{Ak_0}$ | $\boxed{\frac{\alpha\omega\mu(g^\alpha-1)}{A(g^\alpha-\beta)}}\ \frac{m_0}{k_0^h}$ | $\frac{m_0}{c_0+gk_0}$ | $\frac{m_0}{Ak_0}$ | $\frac{m_0}{Ak_0}$ |
| $p_0^h$ | $\frac{A\beta}{\alpha\mu}(k_0^h)^{1-\alpha}$ | $\frac{A}{\alpha}(k_0^h)^{1-\alpha}$ | $\frac{A\beta}{\alpha\mu}(k_0^h)^{1-\alpha}$ | $\frac{A}{\alpha}(k_0^h)^{1-\alpha}$ | $\frac{A\beta}{\alpha\mu}(k_0^h)^{1-\alpha}$ |
| $p_0^l$ | $\frac{A\beta^2}{\alpha\mu}(\frac{1-\alpha}{1-\beta})k_0^h$ | $\frac{A\beta}{\alpha}(\frac{1-\alpha}{1-\beta})k_0^h$ | $\frac{A\beta^2}{\alpha\mu}(\frac{1-\alpha}{1-\beta})k_0^h$ | $\frac{A\beta}{\alpha}(\frac{1-\alpha}{1-\beta})k_0^h$ | $\frac{A\beta}{\alpha}(\frac{1-\alpha}{1-\beta})k_0^h$ |

# Chapter 3

# Natural Vacancy Rate in Hong Kong Housing Market

## 3.1 Introduction

In this chapter, an econometric model is built in order to determine the natural vacancy rate of Hong Kong. Traditional view of housing market operation suggests that there is 'a close connection between excess demand, as reflected in the deviation of the actual vacancy rate from some long-run normal or optimal vacancy rate, and changes in the price of rental housing services.'[1] To be more exact: there is an inverse relation between the vacancy rate and the price of housing services. Through empirical studies, using Hong Kong data from 1983-1996, we confirms the view that the price of housing services is significantly affected by the excess demand in the housing market.

Previous works such as Rosen and smith (1983) and Gabriel and Nothaft (1988) try to determine the natural vacancy rates across different areas in the United States. In contrast to these analyzes, this study employs similar methodology to estimate the natural vacancy rates of different types of houses within Hong Kong.

[1]Rosen and Smith (1983) pp.779.

The outline of this chapter is as follows: model and methodology adopted will be discussed in section 2; the estimated results will be presented in section 3; a conclusion will be given in section 4.

## 3.2 Price-Adjustment Mechanism and Natural Vacancy Rate

Price adjustment mechanism of housing market is typically studied in a stock-flow context. In the short run, stock of house is assumed to be fixed but will change over time. This assumption is based on the fact that new construction, conversion, and demolition take time to process. In the long run the stock of house is affected by the expected rate of return on investment housing market.

When there is an excess demand for houses, say, due to the inflow of immigrants, the actual vacancy rate should move below the long run vacancy rate and this results in an upward pressure on the price of housing services. On the other hand, if there is an decrease in demand on housing, the actual vacancy rate then will move up above the natural vacancy rate and a downward pressure on the price of housing services will be seen. This simply is the basic demand-supply relationship: excess demand (supply) bids up (down) the price. However, why the housing market does not clear? Or, why is there a 'natural' vacancy rate?

Natural vacancy rate, analogous to the natural unemployment rate in the labor market, is 'defined by market factors such as the cost of holding inventory, search costs, the variability of demand, and the cost of recontracting.'[2] Lots of market frictions, namely, high transaction cost, slow response in supply, searching cost, recontracting cost, will slow down the price adjustment of housing services. The natural vacancy rate is defined as the rate at which the price of housing services remain constant.

[2] Rosen and Smith (1983) pp.780.

In this thesis, we follow the hypothesis utilized in Gabriel and Nothaft (1988) that the rate of change of the price of housing services is a function of deviations in the actual vacancy rate from the natural vacancy rate. So the price adjustment mechanism is:

$$\dot{X}_{i,t} = g(V_i^n - V_{i,t-1}) \tag{3.1}$$

where $\dot{X}_{i,t}$ is the growth of the price of housing services at time $t$, $V_i^n$ is the natural vacancy rate, $V_{i,t-1}$ is the actual vacancy rate at time $t-1$, and $g$ is a constant. Note that in Gabriel and Nothaft's model, they tried to determine the natural vacancy rates in different cities of the United States. Therefore, the subscript $i$ in their model referred to different cities across the States; however, what the author trying to do is to determine the natural vacancy rates of different types of houses in Hong Kong. The subscript $i$ here means different types of houses.

As $V_i^n$ is assumed to be constant over time but varies across different types of houses. (3.1) can be rewritten as a pooled cross-section time-series model:

$$\dot{X}_{i,t} = gV_i^n - gV_{i,t-1} + \epsilon_{i,t}; \tag{3.2}$$

though the natural vacancy rates are unobservable, we can compute them from the estimated regression. Assuming the natural vacancy rates of different types of houses are exogenously determined, we can rewrite (3.2) as:

$$\dot{X}_{i,t} = b_0 + \sum_{j=1}^{N-1} b_j d_j - gV_{i,t-1} + \epsilon_{i,t} \tag{3.3}$$

where $d$'s are the type dummy variables, $N$ is the number of types pooled. The natural vacancy rate for type $i$ , $V_i^n$ is calculated as $\frac{\hat{b}_0 + \hat{b}i}{\hat{g}}$; for type $N$, the natural vacancy is $\frac{\hat{b}_0}{\hat{g}}$.

## 3.3 Model Estimation

### 3.3.1 Data

The model is estimated for annual data of Hong Kong private housing sector from 1983 to 1996. The reason of not including the public housing sector is trivial: as the public housing policy in Hong Kong is somewhat a welfare to the low income group, only those families with income lower than the limit can apply. Therefore, the rental of the public house, determined by the government, is usually lower than the market rate. Furthermore, there is always a long list queuing for public houses, vacancy is filled up quickly as searching costs in the private sector do not exist in the public sector. Our model seemingly does not fit the case of the public housing sector in Hong Kong.

According data obtained in the *Hong Kong Property Review*, houses in Hong Kong is divided into 5 types:

*Type A*–with area less than or equal to $39.9\text{m}^2$;

*Type B*–with area between $40\text{m}^2$ and $69.9\text{m}^2$;

*Type C*–with area between $70\text{m}^2$ and $99.9\text{m}^2$;

*Type D*–with area between $100\text{m}^2$ and $159.9\text{m}^2$;

*Type E*–with area not less $160\text{m}^2$.

Rental index is used as the proxy of the price of housing services. The rental index is deflated by consumer price index A with the housing component taken away.

### 3.3.2 Results

Equation (3.2) is estimated using OLS; diagnostic tests suggest that no serial correlation exists but heteroscedasticity is present. White's (1980) heteroscedasticity-consistent estimates of the variance-covariance matrix is computed and used to test hypotheses on the coefficients. The results are shown in table 3.1.

*(Table 3.1 is about here).*

We can tell from the results that a high vacancy rate will hinder the growth in rent. The estimated coefficient of the $V_{i,t-1}$ is -2.8583: with appropriate sign and significant at 1 % level. And the magnitude of the coefficient suggests that, on average, every 1 % increase in the vacancy rate above the natural rate, a drop of 2.86% in the growth rate of rental in the following year will be resulted.

The estimated natural vacancy rates of Type A, Type B, Type C, Type D, and Type E are 3.2857%, 5.3308%, 5.6960%, 6.7755%, and 7.027%, respectively.[3] However, only the dummy for Type A houses, $d_1$, is significant at 5 % level, reflecting differences between the Type A unit and the other types: only Type A's natural vacancy rate is significantly different from the rest. This means that, for a given level of vacancy, renters of Type A houses will experience a moderate increase in rent than the other types' renters. This phenomenon is easy to understand: larger houses usually means higher rent, if one wants to rent a large house, he/she is more willing to search; and the increased willingness to search impede the quick adjustment of the market. Therefore, the natural vacancy rates are higher for Type B, C, D, and E.

## 3.4 Conclusion

This study utilize a pooled cross-section time-series model to estimate the natural vacancy rates of different types of houses using annual data from 1983 to 1996. Research findings are consistent with the traditional view that an inverse relation between price of housing services and natural vacancy rate does exist. Natural vacancy rates of different types of houses are estimated; results indicate that the natural vacancy rate of Type A houses are lower than the other types of houses. And this

[3]The aggregate natural vacancy rate is also determined using the same method. The estimated natural vacancy rate is 4.1285%. The results is consistent with Tse's estimation. Tse (1994), using data from 1980 to 1992, found that the natural vacancy rate of Hong Kong is 4.17%.

can be explained by the increased willingness to search for the renters when he/she is going to pay a great amount for a house.

**Table 3.1:** Results of the OLS Estimation

| | estimated coeifficent | t-ratio | natural vacancy rate % |
|---|---|---|---|
| constant | 20.0842 | 2.9626 | - |
| $Vac_{-1}$ | -2.8583 | -3.3266 | - |
| Type A | -10.6928 | -2.2827 | 3.2857 |
| Type B | -4.8472 | -1.2710 | 5.3308 |
| Type C | -3.8033 | -0.89972 | 5.6960 |
| Type D | -0.71783 | -0.1725 | 6.7755 |
| Type E | - | - | 7.027 |

R-Bar-Squared:0.13923
N: 65

# Chapter 4

# Dynamics of the Property Market in Hong Kong

## 4.1 Introduction

In this chapter, we try to find some relationship between the economy and the housing market. Using data from 1983 to 1996, we find the correlation between the variables to see whether linear relationships exist. Although a high correlation may not represent that there is an economic relationship between the variables, we hope to bring out some insight and benefit future research on the housing market.

Six markets' are examined, namely, the aggregate market, market of Type A houses, market of Type B houses, market of Type C houses, market of Type D houses, and market of Type E houses; types of houses are defined according to their sizes as in *Hong Kong Property Review.*

Eight variables are split into two sets, each set includes four variables. Set one includes growth of real GDP, growth of Consumer Price Index A (CPIA), mortgage rate, and the growth of rental index of the corresponding market; set two includes vacancy of the corresponding market, percentage vacancy of the corresponding market, newly built of the corresponding market; and the newly built divided by the stock of the corresponding market. Each variable in set one will match with the variables in set two. For every match, three correlations will be calculate; first, their contemporaneous correlation; then the correlation with the variable from set one lagged one period; and finally, the correlation with the variable from set two lagged one period. 48 correlations will be computed for each market.

Only those pairs of variables with correlation higher than 0.75 in absolute value will be discussed and the corresponding chart will be plotted.. However, tables of all correlations in different markets will be given at the end of this chapter.

In section two, a description of the pattern of the high correlation pairs is given; a short summary concludes this chapter.

## 4.2 Findings

### 4.2.1 The aggregate market

Only two pairs of variables have correlation larger than 0.75 in absolute value, namely, (1) growth of rental index and newly built and (2) growth of rental index and lagged percentage vacancy; the values of their correlations are 0.8133 and -0.8001, respectively.

*(Table 4.1 is about here)*

We can tell from chart 4.1 that the percentage vacancy and the growth of rental index move countercyclically. The strong negative correlation comes from the trends of the variables: from 1983 to 1989, the percentage vacancy is obviously going down while the growth of rental index is going up. From 1989 to 1995, there is an upward trend in the percentage vacancy while a downward trend is observed in the growth of rental index.

*(Chart 4.1 is about here)*

A strong positive correlation is noted in chart 4.2. The variables, newly built and growth in rental index, move upward from 1983 to 1989. Then, though fluctuate a lot, a downward trend is observed in both variables. The variables move cyclically from 1984 to 1990, then countercyclically until 1993. From 1993 to 1995, they move in the same direction again.

*(Chart 4.2 is about here)*

### 4.2.2 Market of Type A houses

Eight pairs of variables have correlations higher than 0.75 in absolute value. They are:

1. lagged growth in rental index of Type A and vacancy of Type A (-0.7953);
2. lagged growth in rental index of Type A and percentage vacancy of Type A (-0.8005);
3. growth in CPIA and lagged vacancy of Type A (-0.7781);
4. growth in CPIA and lagged percentage vacancy of Type A (-0.8320);
5. growth in CPIA and newly built of Type A (-0.8379);
6. growth in CPIA and newly built of Type A/Stock of Type A (-0.7530);
7. growth in CPIA and lagged newly built of Type A (-0.8693); and
8. growth in CPIA and lagged newly built of Type A/Stock of Type A (-0.8937).

The figures in parentheses are the corresponding correlations.

*(Table 4.2 is about here)*

From chart 4.3, we can tell the vacancy of Type A is moving downward from 1984 to 1988; then the vacancy does not change much in the following three years; from 1991, an upward trend is noted. For the growth of rental index A, it moves up from 1984 until 1990. Then it moves downward from 1990 to 1996. For year-to year movement, no systematic relationship is observed.

*(Chart 4.3 is about here)*

In chart 4.4, a downward trend is observed from1984 to 1991 on the percentage vacancy, then a weak upward trend is followed. Compared with the movement of the growth of rental index A (please refer to the paragraph above), a strong negative correlation is resulted. From 1991, countercyclical movement is detected; however, before 1991, no systematic relationship is observed.

*(Chart 4.4 is about here)*

The correlations of variables from (3) to (8) are all dealt with growth in CPIA. Therefore, we will first discuss the growth in CPIA first; then the 'partners' of it will be examined in turn.

The growth of CPIA first moves down from 1984 to 1986; then it rises from 1986 and reaches its peak at 1991. After that, the value of the variable moves downward gradually.

From chart 4.5, we can tell the vacancy of Type A houses moves down from 1984 to 1992; though it fluctuate up and down during 1984 to 1987, a downward trend is still observed. From 1992, a obvious upward trend is noted. The two series move cyclically for most of the time; however, the relation does not seem to be very strong.

The difference between chart 4.5 and chart 4.6 is not very big. The downward trend of the percentage vacancy of Type A from 1984 to 1992 is more obvious while the upward trend from 1992 is moderate than its counterpart in chart 4.5. The comment of cyclicality on chart 4.5 also applies in this case.

From chart 4.7, we can summarize the trend of newly built of Type A as follows: it moves upward from 1983 to 1985; then downward until 1991. It takes its upward trend again from 1991 to 1994; then a sharp fall in 1995 breaks it. The countercyclicality is so obvious that everyone can tell.

Again, not much different from chart 4.7. As the variable here– newly built of Type A divided by the stock of Type A, moves virtually the same as newly built of Type A, the description of the above paragraph also applies in chart 4.8.

A right shift of newly built of Type A in chart 4.7 results in chart 4.9. The lagged newly built of Type A moves upward from 1984 to 1986; then fluctuates downward until 1992. It moves up again from 1992 to 1995. The cyclical pattern with growth of CPIA from 1984 to 1988 is very strong; however, this relationship is not very clear afterwards.

Again, there is not much different between chart 4.9 and 4.10. The description of chart 4.9 still applies here.

*(Chart 4.5, 4.6, 4.7, 4.8, 4.9, and 4.10 are about here)*

### 4.2.3 Market of Type B houses

Only one pair of variables has correlation higher than 0.75 in absolute value: growth in CPIA and lagged newly built of Type B; a value of 0.7985 is recorded.

*(Table 4.3 is about here)*

The two series almost move together: they both move down first, then up smoothly. Afterwards, fluctuation occurs but the upward trend remains. Between 1988 and 1992, the countercyclicality of the series id very strong; however, for the remaining periods, this relationship is not very clear.

*(Chart 4.11 is about here)*

### 4.2.4 Market of Type C houses

Not a particular pair has high correlation. The pair has the highest correlation (0.7055) in this market is lagged newly built and growth of CPIA.

*(Table 4.4 is about here)*

### 4.2.5 Market of Type D houses

Only one pair of variables has correlation higher than 0.75 in absolute value: growth of rental index of Type D and lagged vacancy of Type D with a correlation of -0.7909

*(Table 4.5 is about here)*

None of the series has a noticeable trend; both of them seem to fluctuate around a constant. Except the period from 1988 to 1991, a strong countercyclicality is observed

*(Chart 4.12 is about here)*

### 4.2.6 Market of Type E houses

Not a particular pair has high correlation. The pair has the highest correlation (-0.6203) is the newly built/stock of Type E and growth of CPIA.

*(Table 4.6 is about here)*

## 4.3 Conclusion

As what we done in this chapter is atheoretical, no systematic conclusion can be drawn. However, from the above findings we note that, among the twelve high correlation pairs, four are relationships between growth of rental index and vacancy while five are relationships between growth of CPIA and newly built. An analysis between growth of rental index and vacancy rate has been done on the last chapter. The relationships between CPIA and newly built still remain unexplored. Further research is needed to find out the truth.

Table 4.1: Correlation for the Aggregate Private Housing Market

| | $Vacancy_t$ | Percentage $Vacancy_t$ | Newly $Built_t$ | Newly $Built_t/Stock_t$ |
|---|---|---|---|---|
| Growth of Real $GDP_t$ | -0.3394 | -0.2532 | 0.09 | 0.2704 |
| Growth of $CPIA_t$ | 0.4789 | 0.3576 | -0.2113 | -0.5316 |
| Mortgage $Rate_t$ | -0.0564 | 0.3090 | -0.4049 | -0.1663 |
| Growth of Rental $Index_t$ | 0.2945 | -0.1441 | *0.8133 | 0.2718 |
| | $Vacancy_t$ | Percentage $Vacancy_t$ | Newly $Built_t$ | Newly $Built_t/Stock_t$ |
| Growth of Real $GDP_{t-1}$ | -0.3754 | -0.5065 | 0.2261 | 0.2967 |
| Growth of $CPIA_{t-1}$ | 0.4666 | 0.4304 | -0.5614 | -0.706 |
| Mortgage $Rate_{t-1}$ | -0.1252 | 0.2499 | -0.5040 | -0.1547 |
| Growth of Rental $Index_{t-1}$ | 0.3566 | -0.086 | 0.1300 | -0.3084 |
| | $Vacancy_{t-1}$ | Percentage $Vacancy_{t-1}$ | Newly $Built_{t-1}$ | Newly $Built_{t-1}/Stock_{t-1}$ |
| Growth of Real $GDP_t$ | -0.1829 | 0.1195 | 0.0808 | 0.4258 |
| Growth of $CPIA_t$ | 0.3723 | -0.0693 | 0.3583 | -0.2272 |
| Mortgage $Rate_t$ | 0.0376 | 0.0248 | 0.1042 | 0.0082 |
| Growth of Rental $Index_t$ | -0.2899 | *-0.8001 | 0.3977 | 0.1649 |

Note: number with * means the correlation between the corresponding variables has an absolute value over 0.75.

**Table 4.2**: Correlation for the Type A houses

| | Vacancy $A_t$ | Percentage Vacancy $A_t$ | Newly Built $A_t$ | Newly Built $A_t$/Stock $A_t$ |
|---|---|---|---|---|
| Growth in Real $GDP_t$ | 0.2789 | 0.2986 | 0.2243 | 0.2099 |
| Growth in $CPIA_t$ | -0.4301 | -0.4032 | *-0.8379 | *-0.7530 |
| Mortgage $Rate_t$ | -0.0130 | 0.2264 | -0.2236 | -0.0695 |
| Growth in Rental Index $A_t$ | -0.6044 | -0.6511 | -0.1888 | -0.3035 |
| | Vacancy $A_t$ | Percentage Vacancy $A_t$ | Newly Built $A_t$ | Newly Built $A_t$/Stock $A_t$ |
| Growth in Real $GDP_{t-1}$ | -0.2472 | -0.1126 | 0.2143 | 0.1982 |
| Growth in $CPIA_{t-1}$ | 0.0500 | -0.0402 | -0.3492 | -0.3101 |
| Mortgage $Rate_{t-1}$ | 0.2890 | 0.4605 | 0.1375 | 0.2598 |
| Growth in Rental Index $A_{t-1}$ | *-0.7953 | *-0.8005 | -0.6169 | -0.6448 |
| | Vacancy $A_{t-1}$ | Percentage Vacancy $A_{t-1}$ | Newly Built $A_{t-1}$ | Newly Built $A_t$/Stock $A_{t-1}$ |
| Growth in Real $GDP_t$ | 0.1090 | 0.1664 | 0.3949 | 0.3783 |
| Growth in $CPIA_t$ | *-0.7781 | *0.8320 | *-0.8693 | *-0.8937 |
| Mortgage $Rate_t$ | -0.4047 | -0.3011 | -0.4864 | -0.4297 |
| Growth in Rental Index $A_t$ | -0.4927 | -0.4664 | 0.0083 | -0.0895 |

Note: number with * means the correlation between the corresponding variables has an absolute value over 0.75.

**Table 4.3**: Correlation for the Type B houses

| | Vacancy $B_t$ | Percentage Vacancy $B_t$ | Newly Built $B_t$ | Newly Built $B_t$/Stock $B_t$ |
|---|---|---|---|---|
| Growth in Real $GDP_t$ | -0.3675 | -0.3823 | -0.0812 | 0.0882 |
| Growth in $CPIA_t$ | 0.5427 | 0.5550 | 0.4654 | 0.2787 |
| Mortgage $Rate_t$ | -0.1462 | -0.1101 | -0.2029 | -0.1824 |
| Growth in Rental Index $B_t$ | 0.3199 | 0.4055 | 0.7486 | 0.7258 |
| | Vacancy $B_t$ | Percentage Vacancy $B_t$ | Newly Built $B_t$ | Newly Built $B_t$/Stock $B_t$ |
| Growth in Real $GDP_{t-1}$ | -0.3104 | -0.2873 | 0.0390 | 0.1764 |
| Growth in $CPIA_{t-1}$ | 0.4302 | 0.3328 | -0.0850 | -0.3578 |
| Mortgage $Rate_{t-1}$ | -0.1915 | -0.1633 | -0.4645 | -0.4722 |
| Growth in Rental Index $B_{t-1}$ | 0.4039 | 0.4326 | 0.6387 | 0.5437 |
| | Vacancy $B_{t-1}$ | Percentage Vacancy $B_{t-1}$ | Newly Built $B_{t-1}$ | Newly Built $B_t$/Stock $B_{t-1}$ |
| Growth in Real $GDP_t$ | -0.2090 | -0.0611 | -0.2053 | -0.0404 |
| Growth in $CPIA_t$ | 0.5419 | 0.5221 | *0.7985 | 0.7289 |
| Mortgage $Rate_t$ | 0.1174 | 0.1612 | 0.3233 | 0.3909 |
| Growth in Rental Index $B_t$ | -0.2459 | -0.2895 | 0.3506 | 0.4766 |

Note: number with * means the correlation between the corresponding variables has an absolute value over 0.75.

**Table 4.4**: Correlation for the Type C houses

| | Vacancy $C_t$ | Percentage Vacancy $C_t$ | Newly Built $C_t$ | Newly Built $C_t$/Stock $C_t$ |
|---|---|---|---|---|
| Growth in Real $GDP_t$ | -0.4770 | -0.5749 | -0.1579 | 0.0160 |
| Growth in $CPIA_t$ | 0.5868 | 0.6090 | 0.3472 | 0.1565 |
| Mortgage $Rate_t$ | 0.06 | 0.3566 | -0.0057 | 0.0811 |
| Growth in Rental Index $C_t$ | 0.0726 | -0.0166 | 0.3509 | 0.4191 |
| | Vacancy $C_t$ | Percentage Vacancy $C_t$ | Newly Built $C_t$ | Newly Built $C_t$/Stock $C_t$ |
| Growth in Real $GDP_{t-1}$ | -0.2209 | -0.0920 | 0.0602 | 0.2818 |
| Growth in $CPIA_{t-1}$ | 0.45 | 0.4531 | -0.1386 | -0.4730 |
| Mortgage $Rate_{t-1}$ | -0.1485 | -0.1260 | -0.3945 | -0.4776 |
| Growth in Rental Index $C_{t-1}$ | 0.3167 | 0.3081 | 0.4562 | 0.4448 |
| | Vacancy $C_{t-1}$ | Percentage Vacancy $C_{t-1}$ | Newly Built $C_{t-1}$ | Newly Built $C_t$/Stock $C_{t-1}$ |
| Growth in Real $GDP_t$ | -0.2634 | -0.0818 | -0.3007 | -0.1347 |
| Growth in $CPIA_t$ | 0.6472 | 0.5927 | 0.7055 | 0.5737 |
| Mortgage $Rate_t$ | 0.2393 | 0.4231 | 0.5210 | 0.5995 |
| Growth in Rental Index $C_t$ | -0.3516 | -0.5274 | 0.1023 | 0.2021 |

**Table 4.5**: Correlation for the Type D houses

| | Vacancy $D_t$ | Percentage Vacancy $D_t$ | Newly Built $D_t$ | Newly Built $D_t$/Stock $D_t$ |
|---|---|---|---|---|
| Growth in Real $GDP_t$ | -0.2787 | -0.0716 | -0.0324 | 0.0837 |
| Growth in $CPIA_t$ | 0.1397 | -0.0935 | -0.2556 | -0.4330 |
| Mortgage $Rate_t$ | 0.5821 | 0.7022 | 0.1169 | 0.1690 |
| Growth in Rental Index $D_t$ | -0.1928 | -0.4160 | 0.4532 | 0.3001 |
| | Vacancy $D_t$ | Percentage Vacancy $D_t$ | Newly Built $D_t$ | Newly Built $D_t$/Stock $D_t$ |
| Growth in Real $GDP_{t-1}$ | -0.2302 | -0.0766 | 0.4243 | 0.4803 |
| Growth in $CPIA_{t-1}$ | 0.0774 | -0.1685 | -0.6058 | -0.7066 |
| Mortgage $Rate_{t-1}$ | 0.0174 | 0.2551 | -0.4172 | -0.3094 |
| Growth in Rental Index $D_{t-1}$ | 0.2514 | -0.1574 | 0.3425 | 0.1238 |
| | Vacancy $D_{t-1}$ | Percentage Vacancy $D_{t-1}$ | Newly Built $D_{t-1}$ | Newly Built $D_t$/Stock $D_{t-1}$ |
| Growth in Real $GDP_t$ | 0.0293 | 0.2424 | -0.0349 | 0.1426 |
| Growth in $CPIA_t$ | 0.2017 | -0.1899 | 0.2680 | 0.0236 |
| Mortgage $Rate_t$ | 0.2177 | 0.1686 | 0.3161 | 0.2620 |
| Growth in Rental Index $D_t$ | *-0.7909 | -0.7088 | -0.1183 | -0.0933 |

Note: number with * means the correlation between the corresponding variables has an absolute value over 0.75.

**Table 4.6**: Correlation for the Type E houses

| | Vacancy $E_t$ | Percentage Vacancy $E_t$ | Newly Built $E_t$ | Newly Built $E_t$/Stock $E_t$ |
|---|---|---|---|---|
| Growth in Real $GDP_t$ | -0.0574 | 0.0226 | 0.0732 | 0.1016 |
| Growth in $CPIA_t$ | 0.4314 | 0.2680 | 0.0088 | -0.0432 |
| Mortgage $Rate_t$ | 0.4920 | 0.5800 | 0.5407 | 0.5551 |
| Growth in Rental Index $E_t$ | -0.5015 | -0.5637 | -0.3981 | -0.4324 |
| | Vacancy $E_t$ | Percentage Vacancy $E_t$ | Newly Built $E_t$ | Newly Built $E_t$/Stock $E_t$ |
| Growth in Real $GDP_{t-1}$ | -0.2281 | -0.1550 | 0.0383 | 0.0716 |
| Growth in $CPIA_{t-1}$ | 0.0673 | -0.0717 | -0.5736 | -0.6203 |
| Mortgage $Rate_{t-1}$ | 0.0992 | 0.1916 | -0.0407 | 0.0096 |
| Growth in Rental Index $E_{t-1}$ | -0.1629 | -0.2937 | -0.0160 | -0.0992 |
| | Vacancy $E_{t-1}$ | Percentage Vacancy $E_{t-1}$ | Newly Built $E_{t-1}$ | Newly Built $E_t$/Stock $E_{t-1}$ |
| Growth in Real $GDP_t$ | 0.1459 | 0.2475 | 0.3740 | 0.4093 |
| Growth in $CPIA_t$ | 0.3478 | 0.1264 | 0.0713 | -0.0380 |
| Mortgage $Rate_t$ | 0.1766 | 0.2120 | 0.3324 | 0.3219 |
| Growth in Rental Index $E_t$ | -0.1685 | -0.2489 | -0.5333 | -0.5097 |

Chart 4.1: Lagged Percentage Vacancy vs growth of rental index

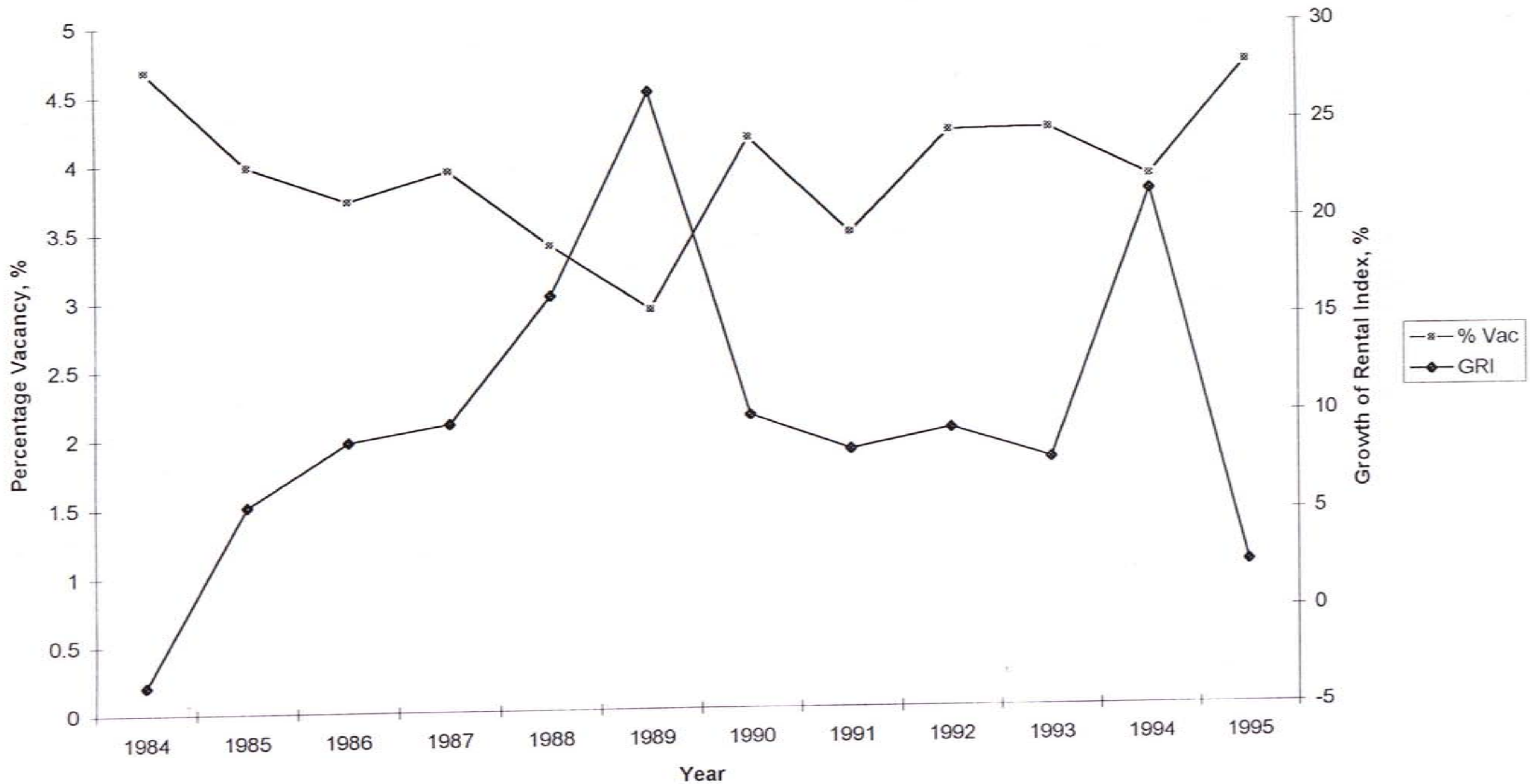


Chart 4.2: Newly Built vs Growth of Rental Index

Chart 4.3: Vacancy of Type At vs Lagged Growth of Rental Index A

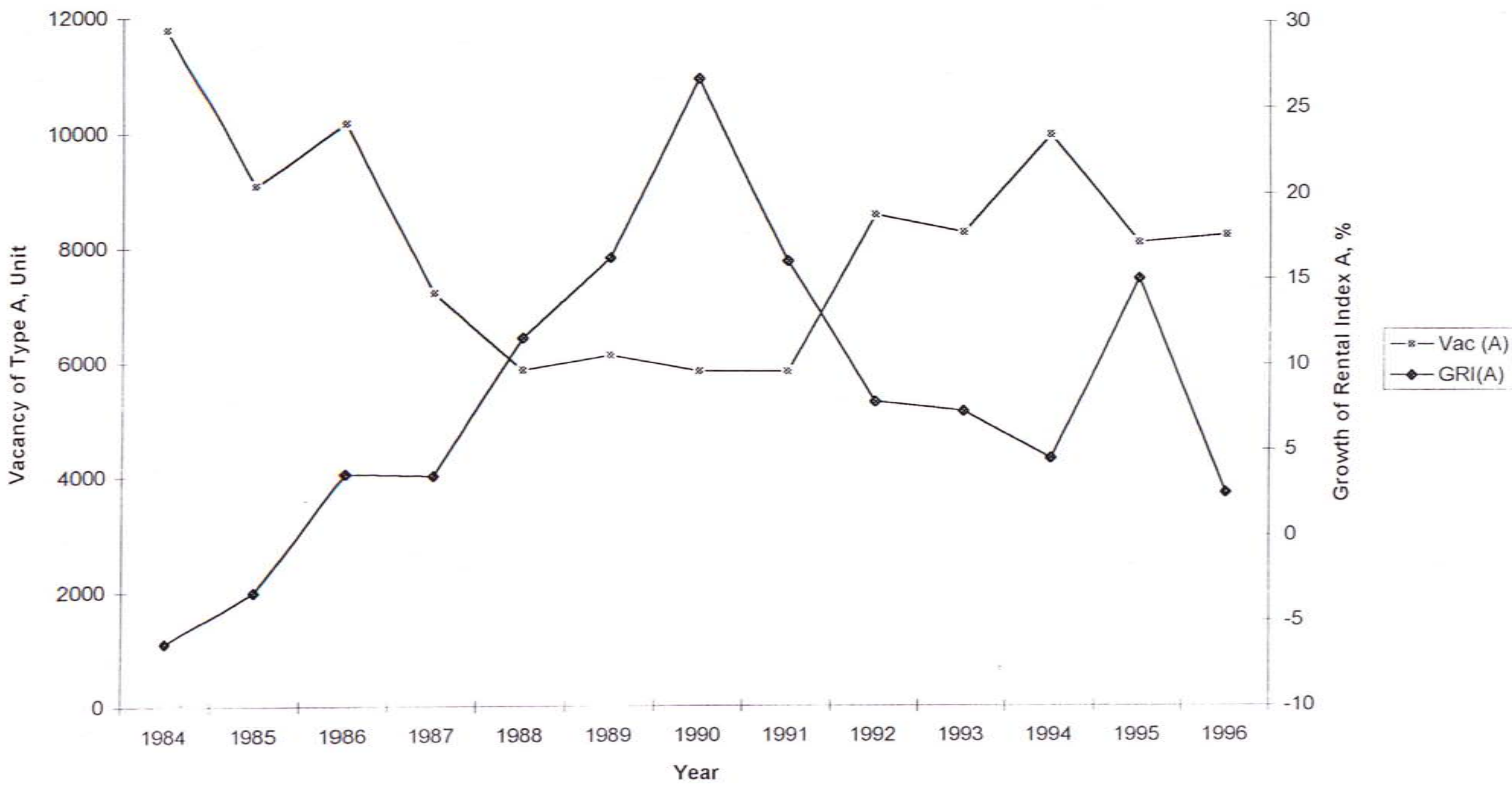


Chart 4.4: Percentage Vacancy of Type A vs Lagged Growth of Rental Index A

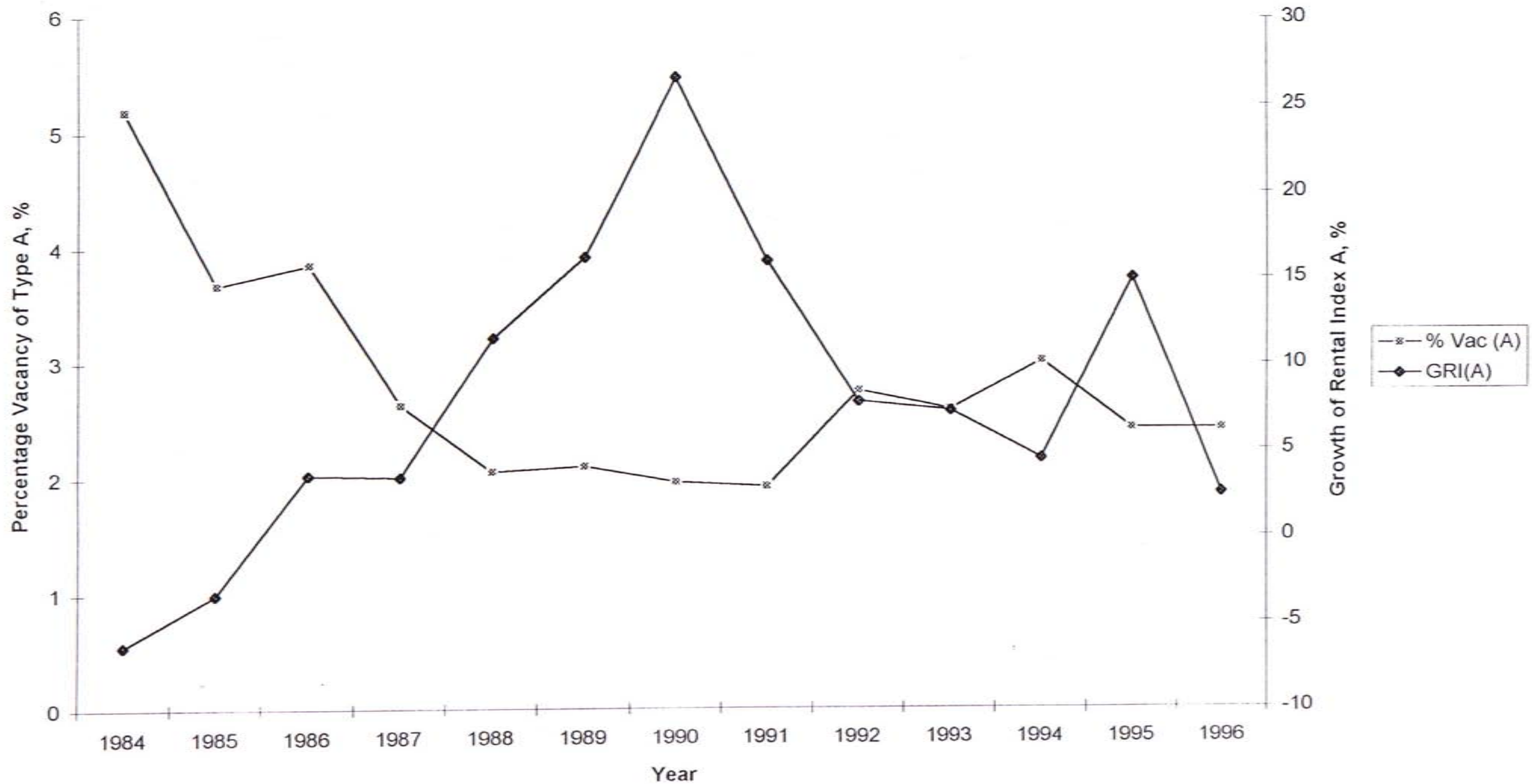


Chart 4.5: Lagged Vacancy of Type A vs Growth of CPIA

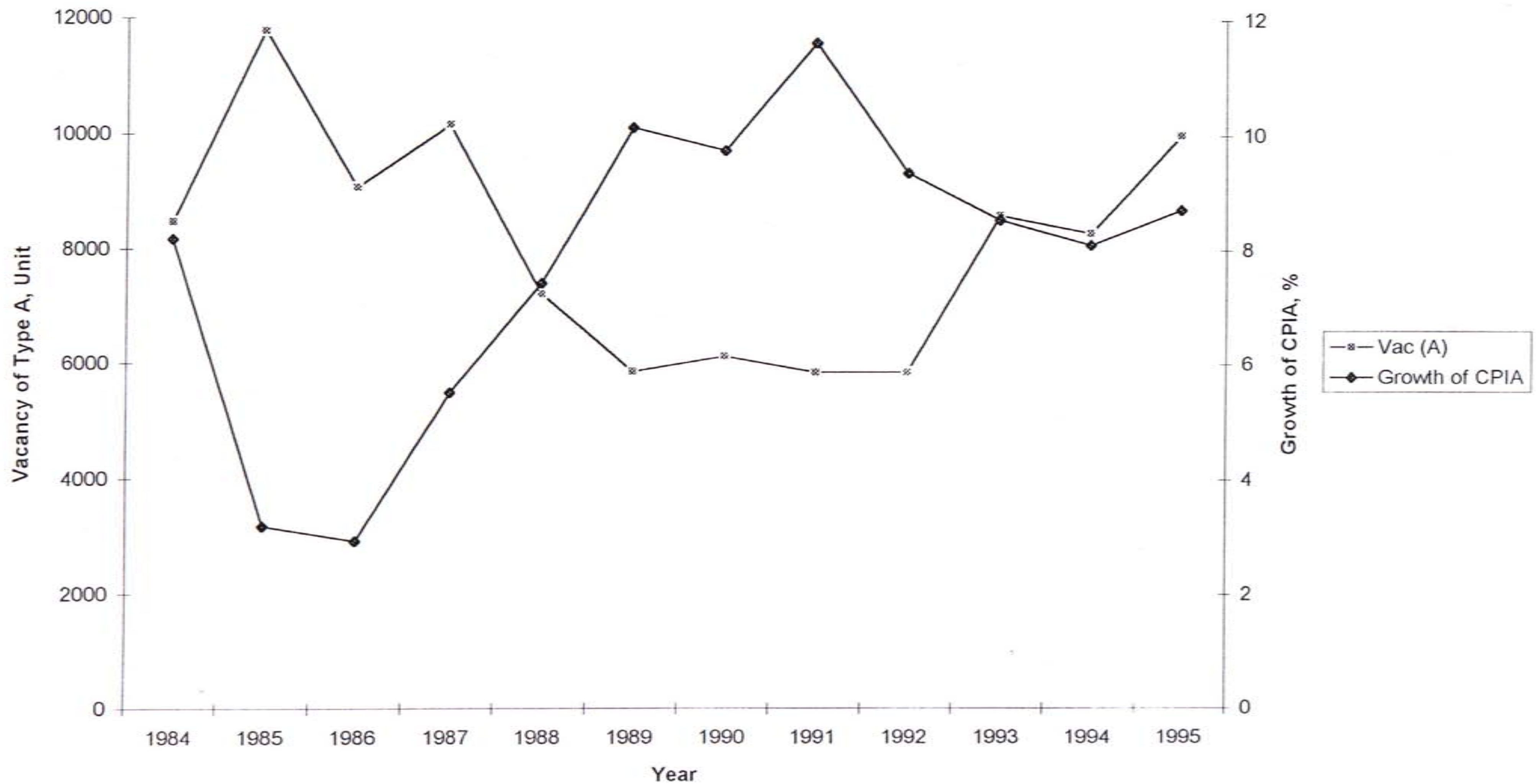


Chart 4.6: Lagged Percentage Vacancy of Type A vs Growth of CPIA

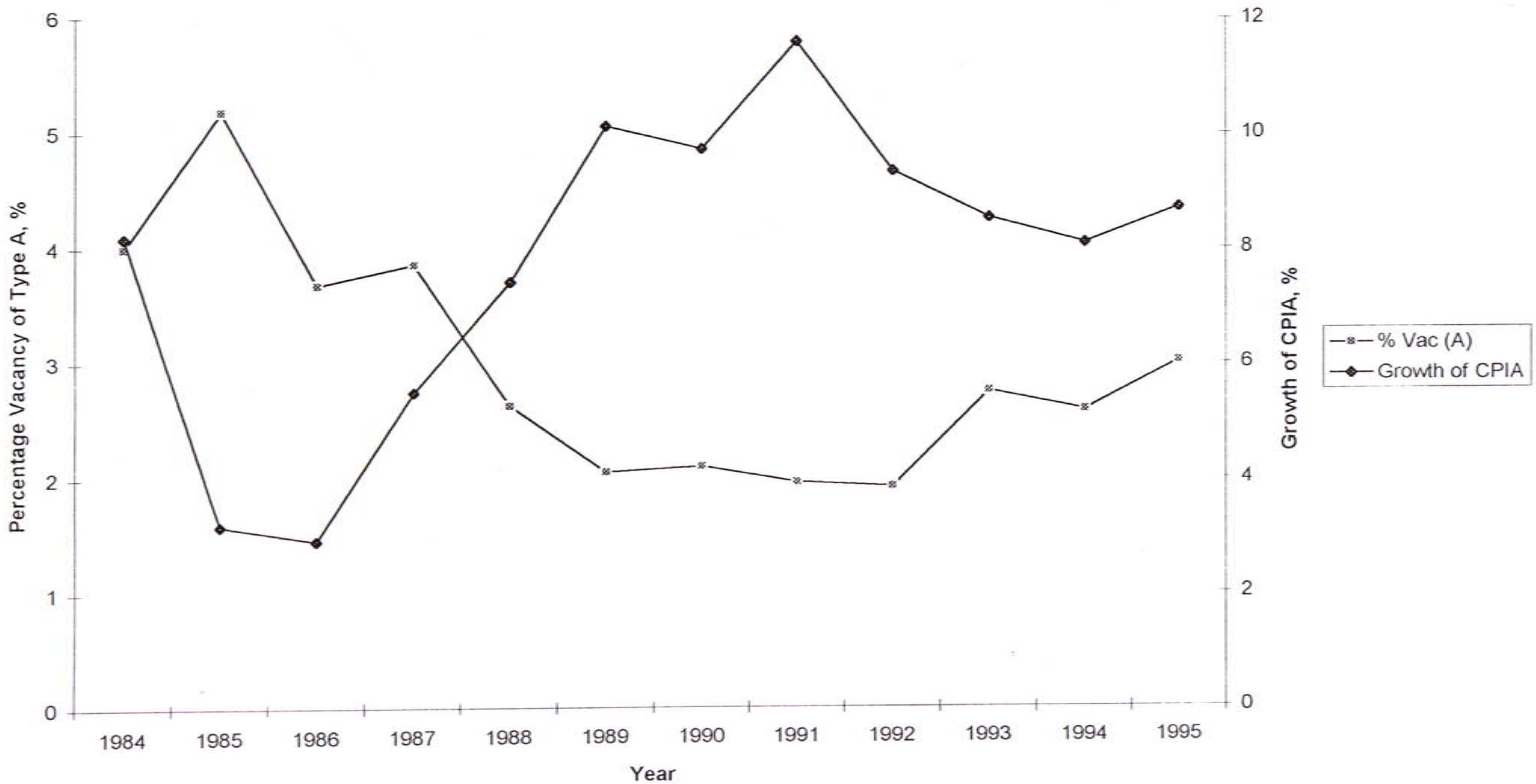


Chart 4.7: Newly Built of Type A vs Growth of CPIA

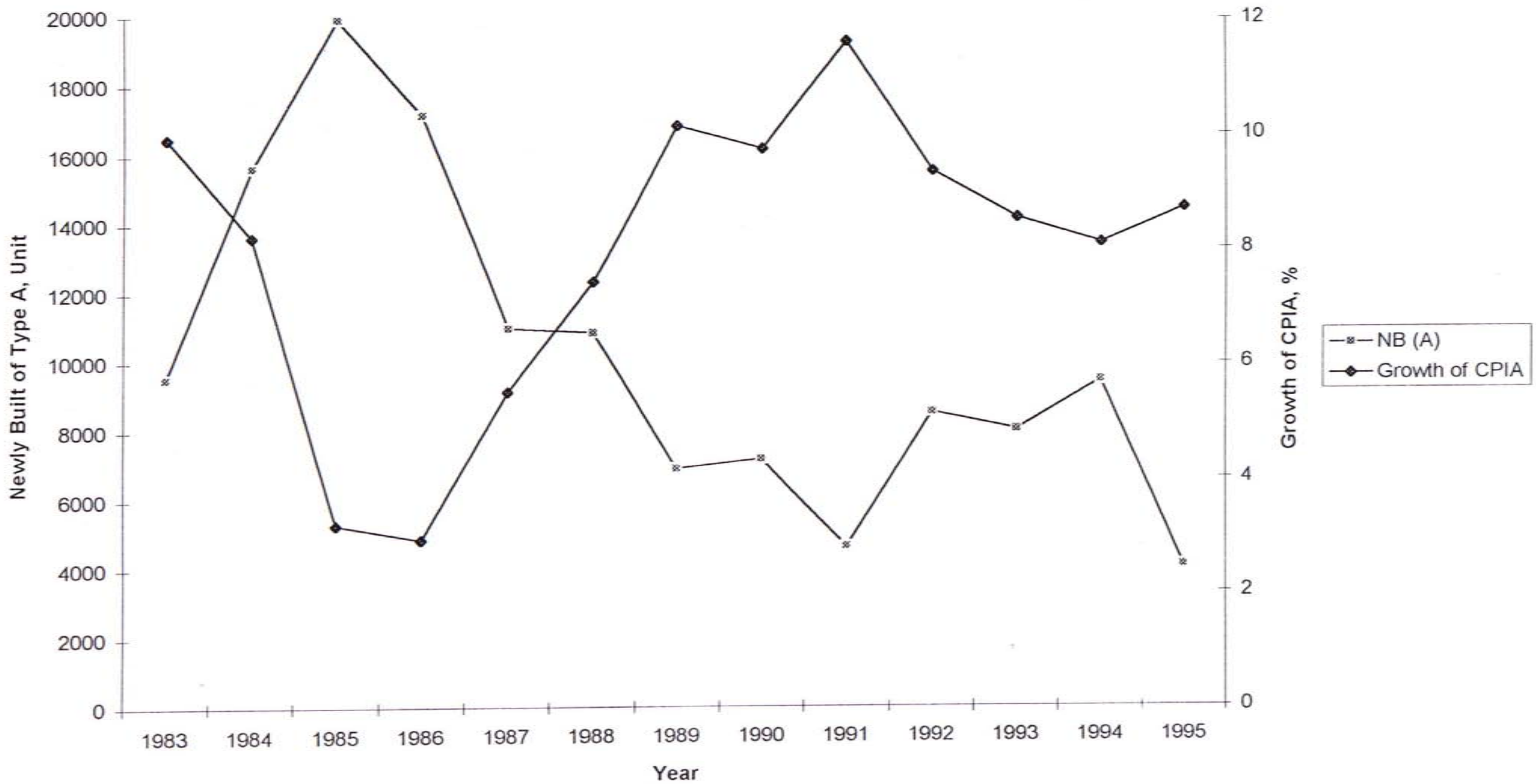


Chart 4.8: Newly Built/Stock of Type A vs Growth of CPIA

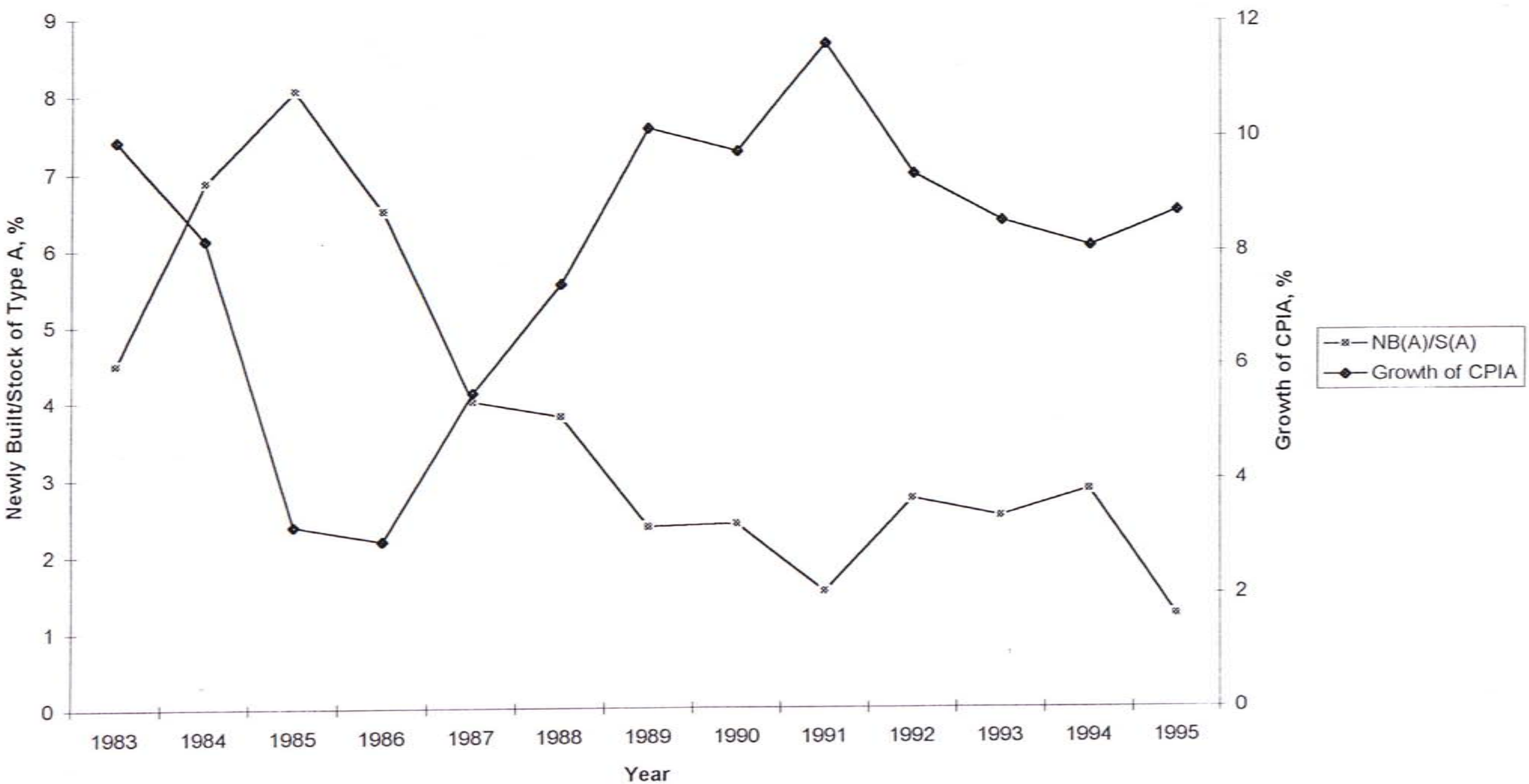


Chart 4.9: Lagged Newly Built of Type A vs Growth of CPIA

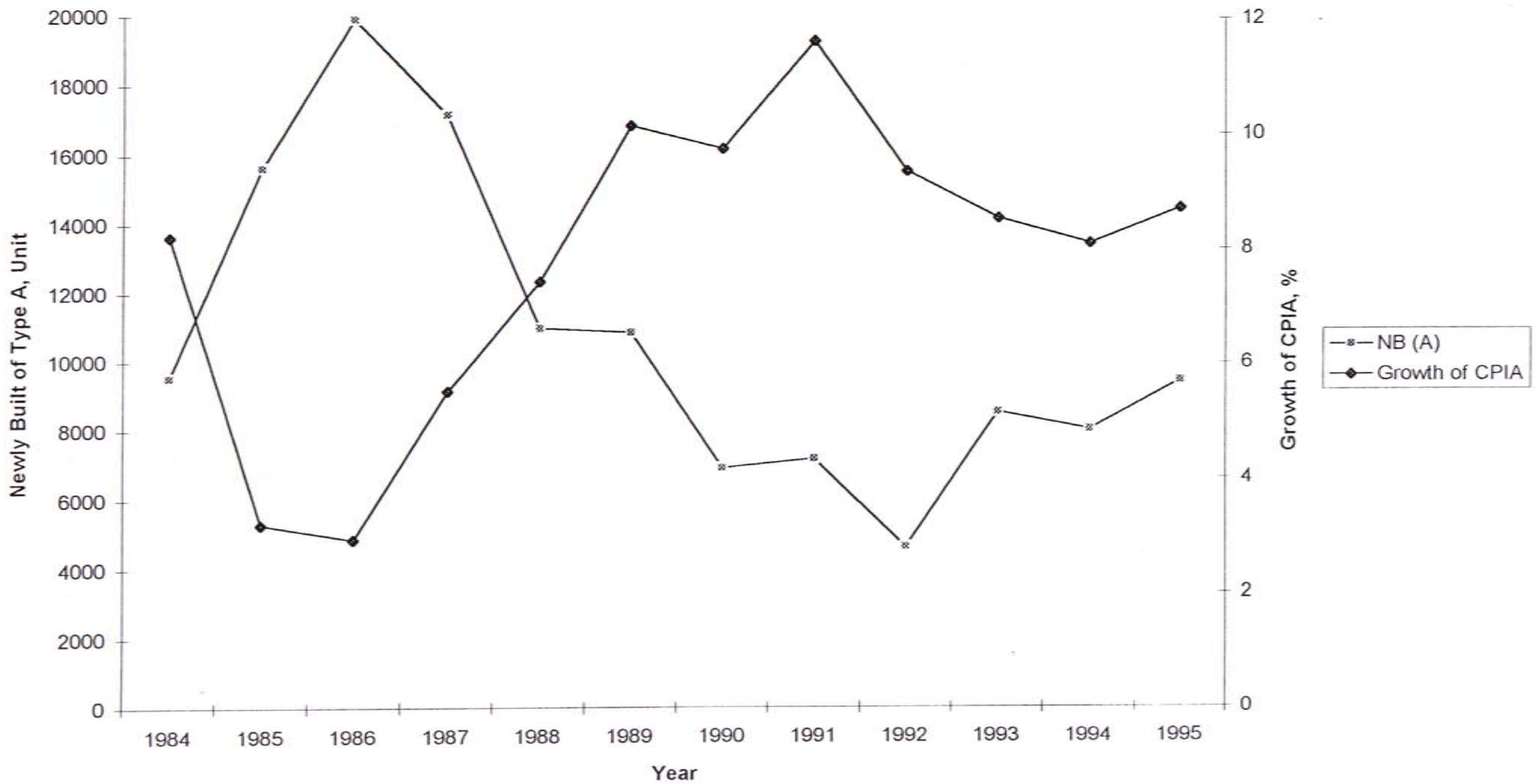
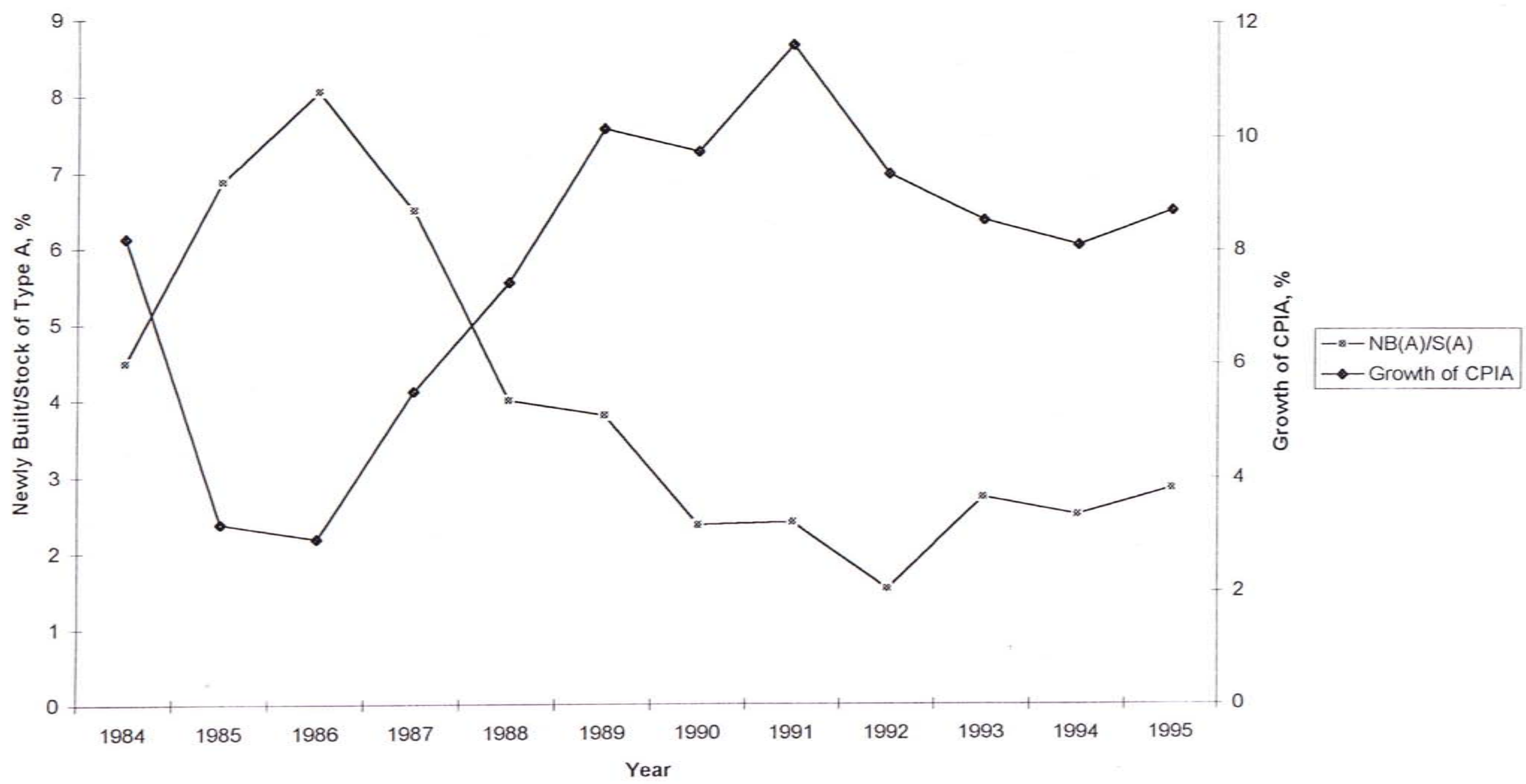


Chart 4.10: Lagged Newly built/Stock of Type A vs Growth of CPIA



## Chart 4.11: Lagged Newly Built of Type B vs Growth of CPIA

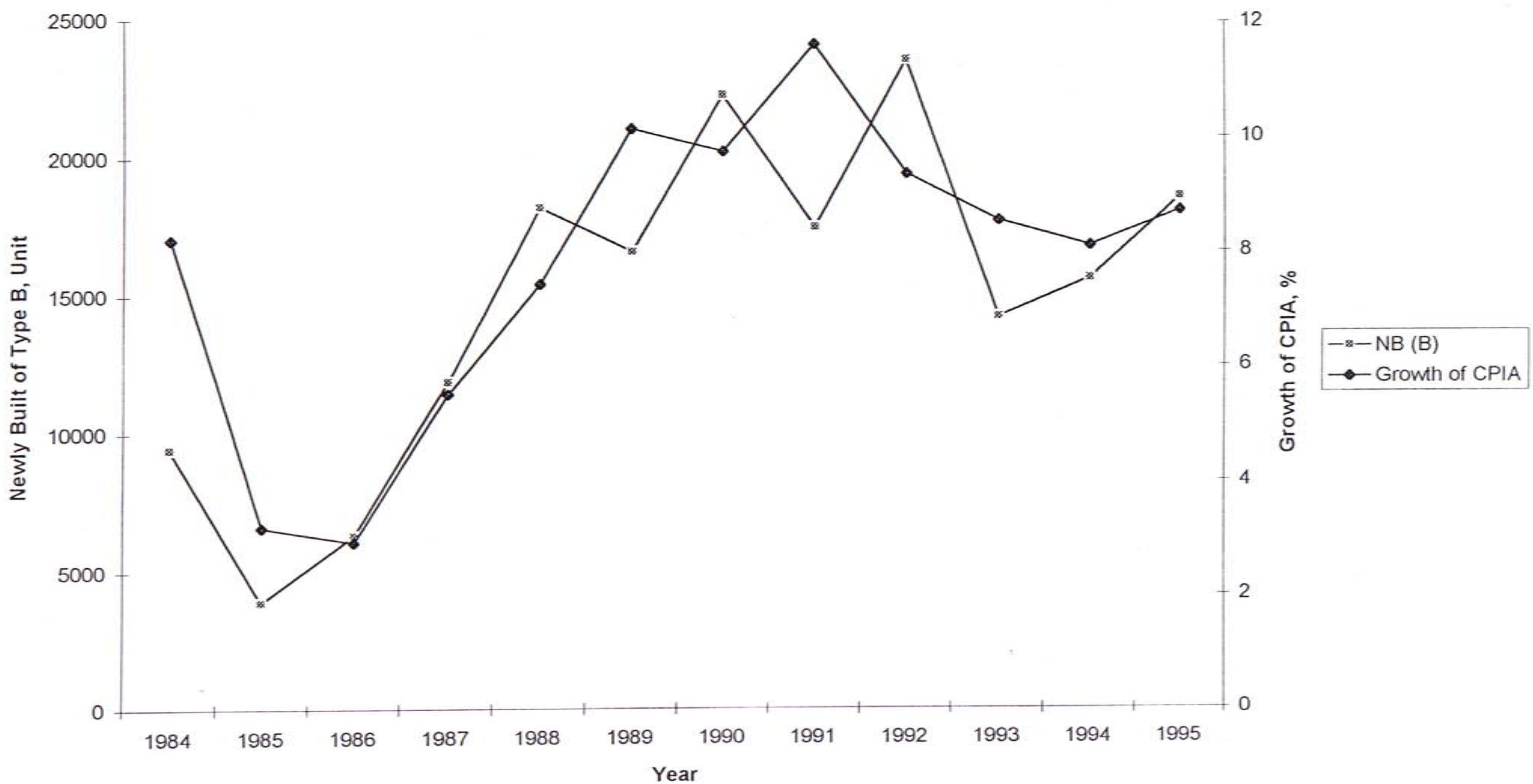
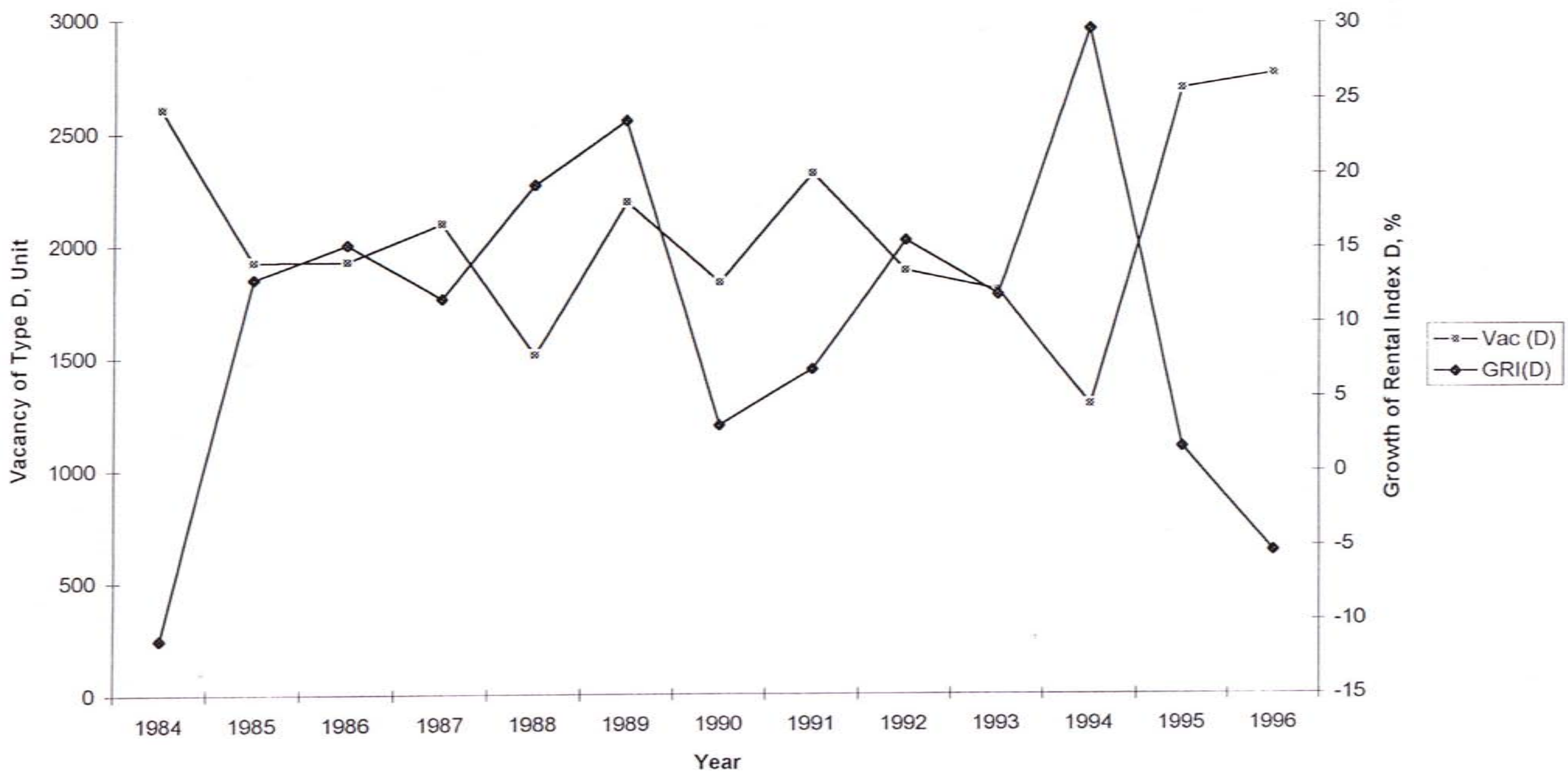


Chart 4.12: Lagged Vacancy of Type D vs Growth of Rental Index D



APPENDICES

**Appendix 2.1: proof of lemma 1**

Rewrite (2.8), we have:

$$h_t = \frac{(k_t^h)^\alpha}{g_h - 1}$$

and $g_h = g_{k^h}^\alpha$.

Using (2.16) and $h_t$ obtained above and the first order conditions (2.9) and (2.13), the path of consumption can easily be shown:

$$c_t = \left[\frac{g_{k^h}(g_h - \beta)}{\alpha\omega\beta^2(g_h - 1)}\right] k_t^h$$

and $g_c = g_{k^h}$.

With (2.9) and (2.16), it is trivial that:

$$p_t^h = \frac{g_{k^h}}{\alpha\beta}(k_t^h)^{1-\alpha},$$

and this implies $g_{p^h} = (g_{k^h})^{1-\alpha}$.

By (2.12) and (2.13), we obtain the difference equation:

$$p_t^l = \beta[\frac{p_{t+1}^l}{g_c} + (\frac{1-\alpha}{\alpha\beta})k_{t+1}^h],$$

solve it recursively, we have,

$$p_t^l = (\frac{1-\alpha}{\alpha})(\frac{g_c}{1-\beta})k_t^h$$

and $g_{p_l} = g_{k^h}$.

By (2.14) and (2.15), we have:

$$A\beta = \frac{\lambda_{2,t}}{\lambda_{1,t+1}} = \frac{\lambda_{2,t}}{\lambda_{1,t}} \cdot \frac{\lambda_{1,t}}{\lambda_{1,t+1}}$$

and

$$\frac{P_{t+1}}{P_t} = \beta\frac{\lambda_{2,t+1}}{\lambda_{2,t}} \cdot \frac{\lambda_{2,t}}{\lambda_{1,t}} = \frac{\beta}{g_c} \cdot \frac{\lambda_{2,t}}{\lambda_{1,t}}.$$

It is trivial that $g_p = A\beta^2 \cdot \frac{1}{g_c^2}$.

Recall that $h_t^m = l_{t+1} - l_t = 0$ at the steady state, the cash-in-advance constraint (2.7) can be written as:

$$\frac{c_t}{k_t} + g_k + \frac{k_{t+1}^h}{k_t} = \frac{m_t}{P_t k_t}$$

at the steady state, we have:

$$\frac{c_{t+1}}{k_{t+1}} + g_k + \frac{k_{t+2}^h}{k_{t+1}} = \frac{m_{t+1}}{P_{t+1} k_{t+1}}.$$

Since

$$(\frac{m_{t+1}}{P_{t+1}k_{t+1}})/(\frac{m_t}{P_t k_t}) = \frac{m_{t+1}P_t k_t}{m_t P_{t+1} k_{t+1}} = \frac{\mu}{g_p g_k} = 1,$$

then

$$\frac{c_t}{k_t}(\frac{g_c}{g_k} - 1) + \frac{k_{t+1}^h}{k_t}(\frac{g_{k^h}}{g_k} - 1) = 0.$$

As we know that $g_c = g_{k^h}$, we have,

$$(\frac{c_t}{k_t} + \frac{k_{t+1}^h}{k_t})(\frac{g_c}{g_k} - 1) = 0.$$

Since the first bracket of the above expression is always positive, the second one has to be zero. As a result, $g_c = g_k$.

**Appendix 2.2: proof of proposition 2**

Market equilibrium requires that all nominal money is held; so, by (2.6),

$$P_t = \frac{m_t}{Ak_t}.$$

And, at the steady state, we have

$$g_p = \frac{\mu}{g_k}.$$

From lemma 1, we have

$$g_p = A\beta^2 \cdot \frac{1}{g_c^2}$$

and

$$g \equiv g_c = g_k.$$

It is trivial that

$$g = \frac{A\beta^2}{\mu}.$$

**Appendix 2.3: proof of lemma 3**

At the steady state, (2.7) becomes:

$$c_0 = \frac{m_0}{P_0} - gk_0 - gk_0^h,$$

as $\frac{m_0}{P_0} = Ak_0$, we have

$$c_0 = Ak_0 - gk_0 - gk_0^h.$$

Combine it with the expression of $c_0$:

$$c_0 = Ak_0 - gk_0 - gk_0^h = \frac{A}{\alpha\omega\mu}(\frac{g^\alpha - \beta}{g^\alpha - 1})k_0^h.$$

After some algebraic transformation we have the initial conditions:

$$\xi_1 k_0 = k_0^h,$$

where $\xi_1 = (A-g)/\left[\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)} + g\right]$.From (2.8), it is trivial that

$$h_0 = \frac{(k_0^h)^\alpha}{g_h - 1}.$$

**Appendix 2.4: proof of lemma 4**

Combine $\lambda_{3,t}$ with (2.26) and (2.24), we have:

$$p_t^h = \frac{\lambda_{3,t}}{\lambda_{3,t+1}}[\frac{1}{\alpha\beta}(k_{t+1}^h)^{1-\alpha}].$$

It is trivial that $\frac{\lambda_{3,t}}{\lambda_{3,t+1}} = g_h$. As $g_h = g_{k^h}^{\alpha}$, we have

$$p_t^h = (\frac{g_{k^h}}{\alpha\beta})(k_t^h)^{1-\alpha}$$

and $g_{p_h} = g_{k^h}^{1-\alpha}$.

By (2.28), we have

$$\lambda_{1,t}\frac{P_{t+1}}{P_t} = \beta\lambda_{2,t+1}$$

and

$$\lambda_{1,t+1}\frac{P_{t+2}}{P_{t+1}} = \beta\lambda_{2,t+2},$$

at the steady state, together with (2.22), we can deduce that

$$\frac{\lambda_{1,t}}{\lambda_{1,t+1}} = \frac{\lambda_{2,t+1}}{\lambda_{2,t+2}} = \frac{\lambda_{2,t}}{\lambda_{2,t+1}} = g_c.$$

By (2.28),

$$c_{t+1} = \frac{\beta}{\lambda_{1,t}g_p};$$

and by (2.24), we have,

$$c_t = \frac{\beta p_t^h}{\lambda_{3,t}g_p g_c};$$

plug the value of $\lambda_{3,t}$ and $p_t^h$ into $c_t$ and make use of (2.8):

$$c_t = \left[\frac{(g_h - \beta)}{\alpha\beta\omega g_p(g_h - 1)}\right] k_t^h,$$

further, we have $g_c = g_{k^h}$.

By (2.25) and (2.26), we have

$$p_t^l = \beta[\frac{\lambda_{1,t}}{\lambda_{1,t+1}}p_{t+1}^l + \frac{\lambda_{3,t+1}}{\lambda_{1,t}}(1-\alpha)(k_{t+1}^h)^{\alpha}];$$

which implies

$$p_t^l = \beta \frac{p_{t+1}^l}{g_c} + (\frac{1-\alpha}{\alpha}) k_{t+1}^h;$$

solving it recursively:

$$p_t^l = (\frac{1-\alpha}{\alpha})(\frac{g_{k^h}}{1-\beta}) k_t^h$$

and we have $g_{p_l} = g_{k^h}$.

By (2.19):

$$P_t = \frac{m_t}{c_t},$$

from this , we have $g_p = \frac{\mu}{g_c}$.

To find $g_k$, we have to make use of (2.20). Similar to problem 1, at the equilibrium, $h_t^m = l_{t+1} - l_t = 0$. So, the budget constraint becomes

$$k_{t+1} + k_{t+1}^h + \frac{m_t}{P_t} = Ak_t$$

which implies

$$g_k + g_{k^h} \frac{k_t^h}{k_t} + \frac{m_t}{P_t k_t} = A$$

Let $\psi_t = \frac{m_t}{P_t k_t}$, then:

$$\frac{\psi_{t+1}}{\psi_t} = \frac{m_{t+1} P_t k_t}{m_t P_{t+1} k_{t+1}} = \frac{\mu}{g_p g_k} = \frac{g_c}{g_k}.$$

If $g_c > g_k$, then $\psi_{t+1} > \psi_t$. That means

$$A - g_k - g_{k^h} \frac{k_{t+1}^h}{k_{t+1}} > A - g_k - g_{k^h} \frac{k_t^h}{k_t};$$

for $g_{k^h}, k^h$,and $k$ are all greater than zero, we have

$$\frac{k_t^h}{k_t} > \frac{k_{t+1}^h}{k_{t+1}}$$

$$\Rightarrow g_k > g_{k^h}$$

$$\Rightarrow g_k > g_c.$$

this contradicts our premise.

On the other hand, if $g_k > g_c$, then $\psi_t > \psi_{t+1}$. That is,

$$\frac{k_{t+1}^h}{k_{t+1}} > \frac{k_t^h}{k_t}$$

$$\Rightarrow g_{k^h} > g_k$$

$$\Rightarrow g_c > g_k$$

which again contradicts the presumption. As a result $g_c = g_k$.

**Appendix 2.5: proof of proposition 5**

By (2.27),

$$\frac{\lambda_{1,t}}{\lambda_{1,t+1}} = A\beta = g_c.$$

As $g \equiv g_c$, we have $g = A\beta$.

**Appendix 2.6: proof of lemma 6**

For the initial condition, consider the budget constraint (2.20) and (2.21) at the equilibrium:

$$k_{t+1} + k_{t+1}^h + \frac{m_t}{P_t} = Ak_t$$

and

$$c_t = \frac{m_t}{P_t};$$

they implies

$$g_k k_0 + g_{k^h} k_0^h + c_0 = Ak_0.$$

As we already know that at time zero $c_0 = \left[\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)}\right] k_0^h$, therefore

$$g_k k_0 + g_{k^h} k_0^h + \left[\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)}\right] k_0^h = Ak_0.$$

And we have

$$\xi_2 k_0 = k_0^h$$

where $\xi_2 = (A - g)/\left[\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)} + g\right]$.

**Appendix 2.7: both consumption and capital are subject to cash-in-advance constraint**

In the present case, the cash-in-advance constraint that the individual is facing in this scenario is

$$c_t + k_{t+1} \leq \frac{m_t}{P_t}.$$

This means that the purchase of both consumption and the capital used in the goods producing sector needs cash in prior.

Again, the constraints are all binding at the equilibrium. The individual is maximizing (2.2) subject to the constraints:

$$k_{t+1}^h + p_t^h h_t^m + p_t^l(l_{t+1} - l_t) + \frac{m_t^d}{P_t} = Ak_t, \tag{A.1}$$

$$c_t + k_{t+1} = \frac{m_t}{P_t} \tag{A.2}$$

and (2.8), the flow of house.

The first order conditions are:

$$\frac{1}{c_t} = \lambda_{2,t}, \tag{A.3}$$

$$\lambda_{3,t} = \beta[\frac{\omega}{h_{t+1}} + \lambda_{3,t+1}] \tag{A.4}$$

$$\lambda_{1,t}p_t^h = \lambda_{3,t} \tag{A.5}$$

$$\lambda_{1,t}p_t^l = \beta[\lambda_{1,t+1}p_{t+1}^l + \lambda_{3,t+1}(1-\alpha)(k_{t+1}^h)^{\alpha}(l_{t+1})^{-\alpha}] \tag{A.6}$$

$$\lambda_{1,t} = \alpha\beta\lambda_{3,t+1}(k_{t+1}^h)^{\alpha-1}(l_{t+1})^{1-\alpha} \tag{A.7}$$

$$\lambda_{2,t} = A\beta\lambda_{1,t+1} \tag{A.8}$$

and

$$\frac{\lambda_{1,t}}{P_t} = \beta\frac{\lambda_{2,t+1}}{P_{t+1}} \tag{A.9}$$

As a rule of thumb, we know that,

$$\lambda_{3,t} = \frac{\omega\beta}{h_t(g_h - \beta)}$$

and this implies $\frac{\lambda_{3,t}}{\lambda_{3,t+1}} = g_h$.

By (A.9), we have

$$\lambda_{1,t}\frac{P_{t+1}}{P_t} = \beta\lambda_{2,t+1}$$

and

$$\lambda_{1,t+1}\frac{P_{t+2}}{P_{t+1}} = \beta\lambda_{2,t+2}$$

at the steady state. With (A.3), these imply

$$\frac{\lambda_{1,t}}{\lambda_{1,t+1}} = \frac{\lambda_{2,t+1}}{\lambda_{2,t+2}} = \frac{\lambda_{2,t}}{\lambda_{2,t+1}} = g_c.$$

From the fact that $g_h = g_{k^h}^{\alpha}$, combining (A.5), (A.7), and the results obtained above, we have

$$p_t^h = (\frac{g_{k^h}}{\alpha\beta})(k_t^h)^{1-\alpha}.$$

and $g_{p^h} = g_{k^h}^{1-\alpha}$.

By (A.3) and (A.9),

$$c_t = \frac{(g_h - \beta)}{\alpha\beta\omega g_p(g_h - 1)} \cdot k_t^h$$

and we have $g_c = g_{k^h}$.

By (A.6) and (A.7), we have

$$p_t^l = \beta\frac{p_{t+1}^l}{g_c} + (\frac{1-\alpha}{\alpha})k_{t+1}^h;$$

solving it recursively,

$$p_t^l = \frac{g_{k^h}(1-\alpha)}{\alpha(1-\beta)}k_t^h$$

and this implies $g_{p^l} = g_{k^h}$.

By (A.2),

$$P_t = \frac{m_t}{c_t + g_k k_t}.$$

Market equilibrium requires all the nominal money is held:

$$m_t^d = m_t.$$

By (A.1) and (A.2), we have

$$k_{t+1}^h + c_t + k_{t+1} = Ak_t.$$

At the steady state, we know that

$$g_{k^h}\left(\frac{k_t^h}{k_t}\right) + \frac{c_t}{k_t} + g_k = A$$

and

$$g_{k^h}\left(\frac{k_{t+1}^h}{k_{t+1}}\right) + \frac{c_{t+1}}{k_{t+1}} + g_k = A.$$

Therefore, we have

$$\left[\frac{c_t}{k_t} + g_{k^h}\left(\frac{k_t^h}{k_t}\right)\right] \cdot \left(1 - \frac{g_c}{g_k}\right) = 0.$$

As the value of the first bracket is always positive, the second one has to be zero. This means $g_c = g_k$.

Again, at the steady state, we have

$$c_t + k_t g_k = \frac{m_t}{P_t}$$

and

$$c_{t+1} + k_{t+1} g_k = \frac{m_{t+1}}{P_{t+1}}.$$

We have $g_p = \dfrac{\mu}{g_c}$.

By (A.8) and (A.9), we know that

$$A\beta = g_c \frac{\lambda_2}{\lambda_1}$$

and

$$g_p = \frac{\beta}{g_c} \frac{\lambda_2}{\lambda_1};$$

this means $g_p = \dfrac{A\beta^2}{g_c^2}$.

To summarize, we have

$$g_c = g_k = g_{k^h} = g_{p^l} \equiv g,$$

$$g_h = g^{\alpha},$$

$$g_{p^h} = g^{1-\alpha},$$

and

$$g_p = \frac{\mu}{g}.$$

As

$$g_p = \frac{\mu}{g} = \frac{A\beta^2}{g^2},$$

that is

$$g = \frac{A\beta^2}{\mu}.$$

The initial values of the endogenous variables are:

$$c_0 = \frac{A\beta(g^{\alpha} - \beta)}{\alpha\omega\mu^2(g^{\alpha} - 1)} \cdot k_0^h,$$

$$p_0^l = \frac{A\beta^2(1-\alpha)}{\alpha\mu(1-\beta)} k_0^h,$$

$$p_0^h = (\frac{A\beta}{\alpha\mu})(k_0^h)^{1-\alpha},$$

and

$$P_0 = \frac{m_0}{c_0 + gk_0}.$$

To derive the initial condition, consider again the expression

$$k_{t+1}^h + c_t + k_{t+1} = Ak_t;$$

combine it with the consumption path that we have obtained above

$$gk_0^h + \frac{A\beta(g^{\alpha} - \beta)}{\alpha\omega\mu^2(g^{\alpha} - 1)} \cdot k_0^h + gk_0 = Ak_0,$$

we have

$$\xi_3 k_0 = k_0^h$$

where $\xi_3 = (A - g) / \left[ \frac{A\beta(g^{\alpha} - \beta)}{\alpha\omega\mu^2(g^{\alpha} - 1)} + g \right]$.

From the above calculation we have already known the path of consumption and the initial condition $\xi_3 k_0 = k_0^h$. Substitute both the expressions into $P_0$, we have

$$P_0 = (A-g) / \left[ \frac{A\beta(g^\alpha - \beta)}{\alpha\omega\mu^2(g^\alpha - 1)} + g \right]^2 \cdot \frac{m_0}{k_0^h}$$

**Appendix 2.8:consumption, capital and capital for house building are subject to cash-in-advance constraint**

In this case, non-durable consumption, capital for construction sector, and capital for goods producing sector are subject to cash-in-advance constraint:

$$c_t + k_{t+1} + k_{t+1}^h \leq \frac{m_t}{P_t}.$$

The individual is maximizing (2.2) under the following constraints:

$$p_t^h h_t^m + p_t^l(l_{t+1} - l_t) + \frac{m_t^d}{P_t} = Ak_t, \tag{A.10}$$

$$c_t + k_{t+1} + k_{t+1}^h = \frac{m_t}{P_t}, \tag{A.11}$$

and the flow of house (2.8).

The first order conditions are:

$$\frac{1}{c_t} = \lambda_{2,t}, \tag{A.12}$$

$$\lambda_{3,t} = \beta[\frac{\omega}{h_{t+1}} + \lambda_{3,t+1}], \tag{A.13}$$

$$\lambda_{1,t}p_t^h = \lambda_{3,t}, \tag{A.14}$$

$$\lambda_{1,t}p_t^l = \beta[\lambda_{1,t+1}p_{t+1}^l + \lambda_{3,t+1}(1-\alpha)(k_{t+1}^h)^\alpha(l_{t+1})^{-\alpha}], \tag{A.15}$$

$$\lambda_{2,t} = \alpha\beta\lambda_{3,t+1}(k_{t+1}^h)^{\alpha-1}(l_{t+1})^{1-\alpha}, \tag{A.16}$$

$$\lambda_{2,t} = A\beta\lambda_{1,t+1}, \tag{A.17}$$

and

$$\frac{\lambda_{1,t}}{P_t} = \beta\frac{\lambda_{2,t+1}}{P_{t+1}}. \tag{A.18}$$

By (A.13), we again have

$$\lambda_{3,t} = \frac{\omega\beta}{h_t(g_h - \beta)}$$

and

$$\frac{\lambda_{3,t+1}}{\lambda_{3,t}} = \frac{1}{g_h}.$$

By (A.16) and (A.17), we can show that

$$\frac{\lambda_{3,t+1}}{\lambda_{1,t+1}} = \frac{A}{\alpha}(k_{t+1}^h)^{1-\alpha}(l_{t+1})^{\alpha-1}.$$

While (A.14) tells us that $\frac{\lambda_{3,t}}{\lambda_{1,t}} = p_t^h$, we can conclude that

$$p_t^h = \frac{A}{\alpha}(k_t^h)^{1-\alpha}$$

and $g_{p^h} = (g_{k^h})^{1-\alpha}$.

Combine (A.12), (A.16), and (2.8), we can show that the consumption path is

$$c_t = \frac{g_{k^h}(g_h - \beta)}{\alpha\omega\beta^2(g_h - 1)}k_t^h$$

and $g_c = g_{k^h}$.

At steady state, we can deduce from (A.17) that

$$\frac{\lambda_{2,t}}{\lambda_{2,t+1}} = \frac{\lambda_{1,t}}{\lambda_{i,t+1}} = g_c.$$

Multiply both sides of (A.15) by $\frac{1}{\lambda_{1,t+1}}$, we have:

$$g_c p_t^l = \beta[p_{t+1}^l + p_{t+1}^h(1-\alpha)(k_{t+1}^h)^{\alpha}]$$

substitute $p_{t+1}^h$ into the above:

$$p_t^l = \frac{\beta}{g_c}[p_{t+1}^l + \frac{A}{\alpha}(1-\alpha)k_{t+1}^h]$$

solve it recursively,

$$p_t^l = \frac{A\beta(1-\alpha)}{\alpha(1-\beta)}k_t^h$$

and we have $g_{p^l} = g_{k^h}$.

For the clearance of the money market, $m_t^d = m_t$ and, at the equilibrium $l_{t+1} - l_t = h_t^m = 0$. Therefore, by (A.10),

$$P_t = \frac{m_t}{Ak_t}$$

and $g_p = \dfrac{\mu}{g_k}$.

Multiply both sides of (A.11) with $\dfrac{1}{k_t}$ , we have

$$\frac{c_t}{k_t} + g_k + \frac{k_{t+1}^h}{k_t} = \frac{m_t}{P_t k_t}.$$

At steady state, we have

$$\frac{c_{t+1}}{k_{t+1}} + g_k + \frac{k_{t+2}^h}{k_{t+1}} = \frac{m_{t+1}}{P_{t+1} k_{t+1}}$$

as well.

It is easy to show that

$$\frac{m_{t+1}}{P_{t+1} k_{t+1}} / \frac{m_t}{P_t k_t} = \frac{\mu}{g_p g_k} = 1;$$

with $g_c = g_{k^h}$ deduced above, we have

$$(\frac{c_t}{k_t} + \frac{k_{t+1}^h}{k_t}) \cdot (\frac{g_c}{g_k} - 1) = 0.$$

Since the first part of the product is always positive, the second part of the above expression must be zero. And this implies that $g_c = g_k$.

By (A.17) and (A.18), we know that

$$A\beta = g_c \frac{\lambda_2}{\lambda_1}$$

and

$$g_p = \frac{\beta}{g_c} \frac{\lambda_2}{\lambda_1}$$

this means $g_p = \dfrac{A\beta^2}{g_c^2}$.

In this case, we have

$$g_c = g_k = g_{k^h} = g_{p^l} \equiv g,$$

$$g_h = g^{\alpha},$$

$$g_{p^h} = g^{1-\alpha},$$

and

$$g_p = \frac{A\beta^2}{g^2}.$$

As

$$g_p = \frac{A\beta^2}{g^2} = \frac{\mu}{g},$$

we can derive

$$g = \frac{A\beta^2}{\mu}.$$

In addition, the initial values of the endogenous variables are:

$$c_0 = \frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)} k_0^h,$$

$$p_0^l = \frac{A\beta(1-\alpha)}{\alpha(1-\beta)} k_0^h,$$

$$p_0^h = \frac{A}{\alpha}(k_0^h)^{1-\alpha}$$

and

$$P_0 = \frac{m_0}{Ak_0}.$$

To derive the initial condition, combine (A.10) and (A.11) with the equilibrium conditions

$$c_t + k_{t+1} + k_{t+1}^h = Ak_t.$$

Substitute the path of consumption into the above expression:

$$\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)} k_0^h + gk_0 + gk_0^h = Ak_0;$$

we have

$$\xi_4 k_0 = k_0^h$$

where $\xi_4 = (A - g) / \left[ \frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)} + g \right]$.

**Appendix 2.9: consumption, capital, capital for house building and purchase on houses are subject to cash-in-advance constraint**

We now turn to the final case: only the purchase of land is not subject to cash-in-advance constraint:

$$c_t + k_{t+1} + k_{t+1}^h + p_t^h h_t^m \leq \frac{m_t}{P_t}.$$

The individual now is maximizing (2.2) with the constraints:

$$p_t^l(l_{t+1} - l_t) + \frac{m_t^d}{P_t} = Ak_t, \tag{A.19}$$

$$c_t + k_{t+1} + k_{t+1}^h + p_t^h h_t^m = \frac{m_t}{P_t} \tag{A.20}$$

and the flow of house (2.8).

The first order conditions are:

$$\frac{1}{c_t} = \lambda_{2,t}, \tag{A.21}$$

$$\lambda_{3,t} = \beta[\frac{\omega}{h_{t+1}} + \lambda_{3,t+1}], \tag{A.22}$$

$$\lambda_{2,t} p_t^h = \lambda_{3,t}, \tag{A.23}$$

$$\lambda_{1,t} p_t^l = \beta[\lambda_{1,t+1} p_{t+1}^l + \lambda_{3,t+1}(1-\alpha)(k_{t+1}^h)^\alpha (l_{t+1})^{-\alpha}], \tag{A.24}$$

$$\lambda_{2,t} = \alpha\beta\lambda_{3,t+1}(k_{t+1}^h)^{\alpha-1}(l_{t+1})^{1-\alpha}, \tag{A.25}$$

$$\lambda_{2,t} = A\beta\lambda_{1,t+1}, \tag{A.26}$$

and

$$\frac{\lambda_{1,t}}{P_t} = \beta\frac{\lambda_{2,t+1}}{P_{t+1}}. \tag{A.27}$$

Once more, solve (A.22) iteratively, we obtain

$$\lambda_{3,t} = \frac{\omega\beta}{h_t(g_h - \beta)}.$$

With the result of $\lambda_{3,t}$, the combination of (A.21) and (A.25) becomes

$$c_t = \left[\frac{g_{k^h}(g_h - \beta)}{\alpha\omega\beta^2(g_h - 1)}\right](k_t^h)$$

and this implies $g_c = g_{k^h}$.

Furthermore, (A.21) and (A.23) become

$$p_t^h = (\frac{g_{k^h}}{\alpha\beta})(k_t^h)^{1-\alpha}$$

and this implies $g_{p^h} = (g_{k^h})^{1-\alpha}$.

By (A.27), we have

$$g_p\lambda_{1,t} = \beta\lambda_{2,t+1}$$

and

$$g_p\lambda_{1,t+1} = \beta\lambda_{2,t+2}.$$

Therefore we have

$$\frac{\lambda_{1,t}}{\lambda_{1,t+1}} = \frac{\lambda_{2,t+1}}{\lambda_{2,t+2}} = \frac{\lambda_{2,t}}{\lambda_{2,t+1}} = g_c.$$

Divide both sides of (A.24) by $\lambda_{1,t+1}$,we obtain

$$p_t^l = \frac{\beta}{g_c}[p_{t+1}^l + \frac{\lambda_{3,t+1}}{\lambda_{1,t+1}}(1-\alpha)(k_{t+1}^h)^\alpha(l_{t+1})^{-\alpha}];$$

from (A.25) and (A.26), we can deduce that

$$\frac{\lambda_{3,t+1}}{\lambda_{1,t+1}} = \frac{A}{\alpha}(k_{t+1}^h)^{1-\alpha}(l_{t+1})^{\alpha-1}.$$

Plug this into the expression of $p_t^l$:

$$p_t^l = \frac{\beta}{g_c}[p_{t+1}^l + \frac{A(1-\alpha)}{\alpha}k_{t+1}^h].$$

Solve this recursively, we have

$$p_t^l = \frac{A\beta(1-\alpha)}{\alpha(1-\beta)}k_t^h$$

and $g_{p^l} = g_{k^h}$

With exactly the same reasons as the cases discussed above, we have

$$P_t = \frac{m_t}{Ak_t}$$

and $g_p = \frac{\mu}{g_k}$.

Further, by (A.26) and (A.27), we have

$$\frac{\lambda_{2,t}}{\lambda_{1,t}} = \frac{A\beta}{g_c} = \frac{g_c g_p}{\beta}$$

this implies $g_p = \frac{A\beta^2}{g_c^2}$.

Divide both sides of (A.20), we obtain

$$\frac{c_t}{k_t} + g_k + \frac{k_{t+1}^h}{k_t} = \frac{m_t}{P_t k_t}$$

at the market equilibrium.

As

$$\frac{m_t}{P_t k_t} / \frac{m_{t+1}}{P_{t+1} k_{t+1}} = \frac{g_p g_k}{\mu} = 1$$

and $g_c = g_{k^h}$, we have

$$\left(\frac{c_t}{k_t} + \frac{k_{t+1}^h}{k_t}\right)\left(1 - \frac{g_c}{g_k}\right) = 0.$$

The first bracket is always positive; so the second one has to be zero. This implies $g_c = g_k$.

In this case, we have

$$g_c = g_k = g_{k^h} = g_{p^l} \equiv g,$$

$$g_h = g^\alpha,$$

$$g_{p^h} = g^{1-\alpha}$$

and

$$g_p = \frac{\mu}{g}.$$

As

$$g_p = \frac{\mu}{g} = \frac{A\beta^2}{g^2},$$

so

$$g = \frac{A\beta^2}{\mu}.$$

The initial values of the endogenous variables are:

$$c_0 = \left[\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)}\right](k_0^h),$$

$$p_0^l = \frac{A\beta(1-\alpha)}{\alpha(1-\beta)}k_0^h,$$

$$p_0^h = (\frac{A\beta}{\alpha\mu})(k_0^h)^{1-\alpha},$$

and

$$P_0 = \frac{m_0}{Ak_0}.$$

At the steady state, the combination of (A.19) and (A.20) becomes:

$$c_t + k_{t+1} + k_{t+1}^h = Ak_t.$$

Plug the path of consumption into the above, we obtain

$$\left[\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)}\right](k_0^h) + gk_0 + gk_0^h = Ak_0;$$

therefore, we have

$$\xi_5 k_0 = k_0^h$$

where $\xi_5 = (A - g)/\left[\frac{A(g^\alpha - \beta)}{\alpha\omega\mu(g^\alpha - 1)} + g\right]$.

**Appendix 2.10: proof of proposition 7**

1. As

$$\mathbb{P}_t = P_t[(1-s_h) + s_h p_t^h],$$

we have

$$\pi_t = \frac{\mathbb{P}_{t+1}}{\mathbb{P}_t} - 1 = g_p \left[ \frac{(1-s_h) + s_h p_{t+1}^h}{(1-s_h) + s_h p_t^h} \right] - 1.$$

With a constant term, the ratio $\frac{(1-s_h) + s_h p_{t+1}^h}{(1-s_h) + s_h p_t^h}$ is always smaller than $\frac{p_{t+1}^h}{p_t^h}$, then $\pi_t < g_p g_p^h - 1$.

2. As

$$\pi_t = g_p \left[ \frac{(1-s_h) + s_h p_{t+1}^h}{(1-s_h) + s_h p_t^h} \right] - 1,$$

if $p_t^h$ grows with time, that is $p_{t+\triangle}^h > p_t^h$, $\pi_t$ increases.

3. As

$$\begin{aligned}\pi_t &= \left\{ g_p \left[ \frac{(1-s_h) + s_h p_{t+1}^h}{(1-s_h) + s_h p_t^h} \right] - 1 \right\} \left( \frac{p_t^h}{p_t^h} \right) \\ &= g_p \left[ \frac{(1-s_h)/p_t^h + s_h g_{p^h}}{(1-s_h)/p_t^h + s_h} \right] - 1\end{aligned}$$

if $p_t^h \to \infty$, $\pi_t$ becomes $g_p g_{p^h} - 1$.

## Appendix 2.11: proof of proposition 8

1. As
$$p^n_{h,t} \equiv \frac{p^h_t}{(1-s_h)+s_h p^h_t} = \frac{p^h_t}{1+s_h(p^h_t-1)}$$
with (2.32) the denominator must be greater than 1. Therefore $p^n_{h,t} < p^h_t$.

2. Referring to (2.34),
$$p^n_{h,t} \equiv \frac{p^h_t}{(1-s_h)+s_h p^h_t}$$
when $p^h_t$ increases, the level of increment in the numerator is greater than that of the denominator (as $s_h < 1$). Therefore, the relative price will increase with housing price.

3. Note that
$$p^n_{h,t}\left(\frac{p^h_t}{p^h_t}\right) = \frac{p^h_t}{(1-s_h)+s_h p^h_t}\left(\frac{p^h_t}{p^h_t}\right)$$
$$= \frac{1}{(1-s_h)/p^h_t + s_h}.$$
As $p^h_t \to \infty$, $p^n_{h,t}$ becomes $\dfrac{1}{s_h}$.

4. As
$$\pi^n_{h,t} \equiv \frac{p^n_{h,t+1}}{p^n_{h,t}} - 1$$
$$= g_{p^h}\left[\frac{(1-s_h)+s_h p^h_t}{(1-s_h)+s_h p^h_{t+1}}\right] - 1$$
and $p^h_t$ increases over time, that is $p^h_{t+1} > p^h_t$, the value of the bracket must be smaller than 1. Therefore, $\pi^n_{h,t} < g_{p^h} - 1$.

5. From
$$\pi^n_{h,t} = g_{p^h}\left[\frac{(1-s_h)+s_h p^h_t}{(1-s_h)+s_h p^h_{t+1}}\right] - 1,$$
it is obvious that when $p^h_t$ increases over time, that is, $p^h_{t+\triangle} > p^h_t$, value of $\pi^n_{h,t}$ will drop.

6. Obviously

$$\begin{aligned}\pi^n_{h,t} &= \left\{ g_{p^h} \left[ \frac{(1-s_h) + s_h p^h_t}{(1-s_h) + s_h p^h_{t+1}} \right] - 1 \right\} \left( \frac{p^h_t}{p^h_t} \right) \\ &= g_{p^h} \left[ \frac{(1-s_h)/p^h_t + s_h}{(1-s_h)/p^h_t + s_h g_{p^h}} \right] - 1;\end{aligned}$$

when $p^h_t \to \infty$, $\pi^n_{h,t} \to 0$.

## BIBLIOGRAPHY

1. Baxter, M. (1996) "Are Consumer Durables Important for Business Cycles," *Review of Economics and Statistics*, 78(1), pp. 147-155.

2. Blank, D. and L. Winnick (1953) "The Structure of the Housing Market," *Quarterly Journal of Economics*, 67, pp. 181-208.

3. Census and Statistics Department, *Hong Kong Monthly Digest of Statistics*. Hong Kong : Hong Kong Government Printer. Various Issues.

4. de Leeuw, F. and N. Ekanem (1971) "The Supply of Rental Housing," *American Economic Review*, 61(5), pp. 806-817.

5. Easterly, W., R. King, R. Levine, and S. Rebelo (1993) "Policy, Technology Adoption and Growth," in R. Solow and L. Pasinetti (eds.), *Economic Growth and the Structure of Long Term Development*. London: Macmillian Press.

6. Eubank, A. and C. Sirmans (1979) "The Price Adjustment Mechanism for Rental Housing in the United States," *Quarterly Journal of Economics*, 93(1), pp. 163-168.

7. Fair, R. (1972) "Disequilibrium in Housing Models," *Journal of Finance*, 27(2), pp. 207-221.

8. Gabriel, S. and F. Nothaft (1988) "Rental Housing Markets and the Natural Vacancy Rate," *AREUEA Journal*, 16(4), pp. 417-429.

9. Government of Hong Kong Special Administration Region, *Economic Background*. Hong Kong: Hong Kong Government Printer. Various Issues.

10. Greenwood, J. and Z. Hercowitz (1991) "The Allocation of Capital and Time over the Business Cycle," *Journal of Political Economy*, 99, pp. 1188-1214.

11. Greenwood, J., Z. Hercowitz, and P. Krusell (1997) "Long-Run Implication of Investment-Specific Technological Change," *American Economic Review*, 87, pp. 342-362.

12. Information Services Department, (1997) *Hong Kong 1997.* Hong Kong: Hong Kong Government Printer.

13. Judge, G., R. Hill, W. Griffiths, H. Lutkepohl, and T. C. Lee (1988) *Introduction to the Theory and Practice of Econometrics.* 2nd ed. New York: Wiley.

14. King, R., C. Plosser, and S. Rebelo (1988) "Production, Growth and Business Cycles: I. the Basic Neoclassical Model," *Journal of Monetary Economics*, 21, pp. 195-232.

15. Leung, K.Y. (1998) "Economic Growth and Increasing Housing Price Puzzle," Working Paper. The Chinese University of Hong Kong.

16. Li, H. L. (1997) *Development Appraisal of Land in Hong Kong.* Hong Kong: The Chinese University Press.

17. Olsen, E. (1969) "A Competitive Theory of the Housing Market," *American Economic Review*, 59(4), pp. 612-622.

18. Pesaran, M. and B. Pesaran (1997) *Working with Microfit 4.0: Interactive Econometric Analysis.* Oxford: Oxford University Press.

19. Ramanathan, R. (1992) *Introductory Econometrics with Application.* 2nd ed. New York: The Dryden Press.

20. Rating and Valuation Department, *Hong Kong Property Review.* Hong Kong: Hong Kong Government Printer. Various Issues.

21. Rebelo, S. (1991) "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, 99, pp. 500-521.

22. Renaud, B., F. Pretorius, and B. Pasadilla (1997) *Markets at Work: Dynamics of the Residential Real Estate Market in Hong Kong.* Hong Kong: Hong Kong University Press.

23. Sargent, T. (1987) *Dynamic Macroeconomic Theory.* Cambridge: Harvard University Press.

24. Smith, L. (1969) "A Model of the Canadian Housing and Mortgage Markets," *Journal of Political Economy*, 77(5), pp. 795-816.

25. Smith, L. (1974) "A Note on the Price Adjustment Mechanism for Rental Housing," *American Economic Review*, 64(3), pp.478-481.

26. Stokey, N. and R. Lucas, with E. Prescott (1989) *Recursive Methods in Economic Dynamics.* Cambridge: Harvard University Press.

27. Tse, Y. C. (1994) *Real Estate Economics: Theory and Policy with Reference to Hong Kong, Singapore and Taiwan.* Hong Kong: EIA Publishing Ltd.

28. White, H. (1980) "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica*, 48, pp. 817-838.